

# Berlin-Poznań Seminar on Discrete Mathematics

Friday, 4 June 2010

Technische Universität Berlin

The Berlin-Poznań Seminar is a joint seminar organized by the three Berlin Universities (FU, HU and TU Berlin) and the Adam Mickiewicz University in Poznań. The topics include discrete mathematics and algorithms, enumerative, extremal and probabilistic combinatorics. It is supported by DFG within the Research Training Group (Graduiertenkolleg) "Methods for Discrete Structures" and the Berlin Mathematical School and by the Adam Mickiewicz University in Poznań.

## Programme

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|---------------|----------------------------------------------------------------------------------------|
| 09:00 – 09:10 | Opening<br>Welcome address by Günter M. Ziegler                                        |
| 09:10 – 10:00 | Tomasz Łuczak<br>The structure of dense $H$ -free graphs                               |
| 10:00 – 10:40 | Tibor Szabó<br>Box-game revisited                                                      |
| 10:40 – 11:20 | Katarzyna Rybaczuk<br>On relations between $G(n, p)$ and the random intersection graph |
| 11:20 – 13:00 | Lunch break                                                                            |
| 13:00 – 13:40 | Hiệp Hàn<br>Weak quasi-randomness for uniform hypergraph                               |
| 13:40 – 14:20 | Rafał Witkowski<br>Graph multicoloring in frequency assignment problem                 |
| 14:20 – 15:00 | Kolja Knauer<br>Lattices and polytopes from graphs                                     |
| 15:00 –       | Closing                                                                                |

## Location

Technische Universität Berlin  
Institut für Mathematik  
Straße des 17. Juni 136, D-10623 Berlin  
Lecture Room MA 415

## Abstracts

### Tomasz Łuczak

“The structure of dense  $H$ -free graphs”

In the talk we state a few questions, results, and conjectures concerning the structure of (large) graphs on  $n$  vertices with large minimum degree which contain no copies of a given (small) graph  $H$ . Most of them were inspired by a series of classical papers of Hungarian mathematicians from early seventies. This is a joint work with Stéphan Thomassé.

### Tibor Szabó

“Box-game revisited”

The Box-game is a positional game where two players, Maker and Breaker alternately throw balls of their color into one of  $k$  boxes of various sizes. In each round Maker throws  $m$  red balls while Breaker throws one blue ball. Maker wins if he fills up a box with red balls, otherwise Breaker wins. This game was defined by Chvatal and Erdos in 1978 as an auxiliary game for their analysis of the biased connectivity game on the complete graph. In the talk we reconsider this classic. Joint work with Dan Hefetz, Michael Krivelevich, and Milos Stojakovic.

### Katarzyna Rybaczyk

“On relations between  $G(n, p)$  and the random intersection graph”

In a random intersection graph  $G(n, m, p)$  to each vertex  $v$  from the vertex set  $V$  ( $|V| = n$ ) we assign a set of its features  $D_v$  by choosing independently each feature with probability  $p$  from the feature set  $W$  ( $|W| = m = n^\alpha$  for some constant  $\alpha$ ). Then we connect vertices  $v, v' \in V$  by an edge if and only if sets  $D_v$  and  $D_{v'}$  intersect. In many aspects  $G(n, m, p)$  differ much from the widely studied model with independent edges. However known results suggest, that for certain parameters  $\hat{p}$ ,  $n$  and  $m$  one can expect that there is some relation between  $G(n, m, p)$  and  $G(n, \hat{p})$ . So far it has been proved by A. Fill, E. R. Scheinerman and K. B. Singer–Cohen, that both models have asymptotically the same properties for  $\alpha > 6$  and  $\hat{p}$  approximately equal to the edge probability in  $G(n, m, p)$ . In the talk there will be presented the extension of the above result to smaller values of  $\alpha$ . Moreover it will be shown that in some cases it is possible to obtain interesting results using relations between  $G(n, m, p)$  and  $G(n, \hat{p})$  despite the fact that two models are not equivalent.

## Hiệp Hàn

“Weak quasi-randomness for uniform hypergraph”

Quasi-randomness is a concept that studies deterministic objects that behave like random objects do. It was first introduced by Thomason and in their seminal paper Chung, Graham and Wilson introduced several graph properties that all turned out to be equivalent. In the talk we will consider some of these properties and focus on their generalisations to uniform hypergraphs. This is a joint work with David Conlon, Yury Person and Mathias Schacht.

## Rafał Witkowski

“Graph multicoloring in frequency assignment problem”

Given a graph  $G$  and a demand function  $p : V(G) \rightarrow \mathbb{N}$ , a proper  $n$ - $[p]$ coloring is a mapping  $f : V(G) \rightarrow \{1, \dots, n\}$  such that  $|f(v)| \geq p(v)$  for any vertex  $v \in V(G)$  and  $f(v) \cap f(u) = \emptyset$  for any pair of adjacent vertices  $u$  and  $v$ . The least integer  $n$  for which a proper  $n$ - $[p]$ coloring exists,  $\chi_p(G)$ , is called the *multichromatic number* of  $G$ . Finding the multichromatic number of induced subgraphs of the triangular lattice (called *hexagonal graphs*) has important applications in cellular networks. The *weighted clique number* of a graph  $G$ ,  $\omega_p(G)$ , is the maximum weight of a clique in  $G$ , where the *weight* of a clique is the total demand of its vertices. McDiarmid and Reed conjectured that  $\chi_p(G) \leq (9/8)\omega_p(G) + O(1)$  for triangle-free hexagonal graphs. During the talk we provide an algorithm to find a 7- $[3]$ coloring of triangle-free hexagonal graphs, which implies that  $\chi_p(G) \leq (7/6)\omega_p(G) + O(1)$ . We also show how to prove McDiarmid and Reed conjecture for large and wellknown subclass of triangle-free hexagonal graphs.

## Kolja Knauer

“Lattices and polytopes from graphs ”

We describe a method to obtain a lattice structure on several objects coming from digraphs. Some of the instances that can be identified by this unifying approach are: integral tensions, configurations of chip-firing-games and flow in planar digraphs.

Moreover every lattice arising that way can be seen as the set of (integral) points of a polyhedron. This embedding in Euclidean space reflects the order relation. A particular instance of this phenomenon is the Order polytope, which is the convex hull of the ideals of a poset.