

Functional Analysis I

7th problem sheet

Please return your answers in the tutorials on June, 5th / 6th
(together with the answers to the 6th problem sheet)

As there are no exercises and tutorials this week, this is a reduced problem sheet.

Problem 1:

5 pt.

Let $X = l^2(\mathbb{N})$ and consider the operator

$$T : X \rightarrow X, \quad x = (x_n) \mapsto Tx := \left(\frac{x_n}{n}\right).$$

Show that T is compact. For which $\lambda \in \mathbb{C}$ does the operator $(T - \lambda)^{-1}$ not exist?
Show that for these λ the dimension of $\ker(T - \lambda)$ is finite. What's about $\lambda = 0$?

Problem 2:

5 pt.

Let X be a Banach space and let T be a compact operator in X , $T \in K(X)$.

- (i) Show that $\ker(Id - T)$ is finite dimensional.
- (ii) If $Id - T$ is bijective, then $(Id - T)^{-1}$ is continuous.
Remark: This is true for all bounded linear operators, but much harder to show then.
- (iii) Assume that X is infinite dimensional. Show that

$$d(Id, K(X)) = \inf_{T \in K(X)} \|Id - T\| = 1.$$

Hint: $T = Id - (Id - T)$, Neumann series.