Tutorials on April, 24th / 25th

Problem 1:
Let X be the space of sequences for which only a finite number of entries is non-zero. Show that $(X, \|\cdot\|_1)$ is a normed but not a Banach space.

Problem 2:
A normed space $(X, \|\cdot\|)$ is called **strictly convex** if $\|x+y\| < 2$ holds for all $x, y \in X$, $\|x\| = \|y\| = 1$, $x \neq y$. Show that the following are equivalent:

(i) X is not strictly convex.

(ii) There exist linear independent $x, y \in X$ with $\|x+y\| = \|x\| + \|y\|$.

(iii) The unit sphere $S = \{x \in X \mid \|x\| = 1\}$ contains a segment of a straight line.

Problem 3:
Show that $C([0,1])$ is not strictly convex.

Problem 4:
Consider the sequences of functions

$$x_n(t) := t^n - t^{n+1}, \quad y_n(t) := n^{3/2}(t^n - t^{n+1}), \quad t \in [0,1].$$

Are these sequences bounded or convergent in $C([0,1])$ or $L^p([0,1])$, $1 \leq p \leq 2$?