

## Exercise sheet 9

**Tutorial** (17.06.2013)

**T18.** Find a conservation law for the equation

$$\partial_{11}u - \partial_{22}u - \partial_{33}u = 0.$$

**T19.** Prove that the generalized vector fields  $\mathbf{v}_1 := (x_1\partial_2u - 1)\partial$  and  $\mathbf{v}_2 := \partial_1u\partial$  define generalized symmetries of the equation

$$\partial_{222}u + u\partial_2u + \partial_1u = 0.$$

**Homework** (Deadline: 27.06.2013, 12:30 at the beginning of the lecture)

*Remark:* These exercises are to be solved completely by hand. Nevertheless, it is a good idea to check the results via Maple etc.

**H22.** (13 points) Let  $F \in \mathcal{A}$  and consider a differential operator acting on  $F$ :

$$\mathcal{D}[F] := \sum_{|\ell| \geq 0}^{\ell} P_{\ell} \mathbb{D}_{\ell} F, \quad P_{\ell} \in \mathcal{A}.$$

The formal adjoint of  $\mathcal{D}$  is the differential operator defined by

$$\mathcal{D}^*[F] := \sum_{|\ell| \geq 0}^{\ell} (-\mathbb{D})_{\ell} (P_{\ell} F), \quad (-\mathbb{D})_{\ell} := (-1)^{|\ell|} \mathbb{D}_{\ell}.$$

**a)** Prove that  $\mathcal{D}^*$  satisfies

$$\int_X F \mathcal{D}[G] \, dx = \int_X G \mathcal{D}^*[F] \, dx,$$

for every  $F, G \in \mathcal{A}$  which vanish when  $u \equiv 0$ , every  $X \subset \mathbb{R}^m$  and every function  $u = f(x)$  of compact support in  $X$ .

**b)** Prove that

$$\mathbb{E}[F \mathcal{D}[G]] = \mathbb{E}[G \mathcal{D}^*[F]]$$

for every pair of functions  $F, G \in \mathcal{A}$ . Here  $\mathbb{E}$  is the Euler operator.

**H23.** (17 points) Prove that the generalized vector field

$$\mathbf{v} := \left( 4\partial_{222}u - 6u\partial_{22}u - 6(\partial_2u)^2 + 3u^2\partial_2u \right) \partial$$

defines a generalized symmetry of the equation

$$\partial_{22}u - u\partial_2u - \partial_1u = 0.$$

### Homework policy

Homework assignments are due weekly at the beginning of the Tutorial (Monday, 14:15). They have to be turned in directly to the lecturer. No homework will be accepted after the deadline has passed.

Homework assignments can be to be solved in groups of two people.

To get the Übungsschein, you need to satisfactorily complete 60% of the homework assignments and to present the solution of one homework exercise at the blackboard in the tutorial.