Example: Berlin airlift

- four quarters of year, cargo to be transported in each:
  100, 150, 150, 200 units of cargo, where
  1 unit ≙ 1 plane carriage unit ≙ 14 t
- each airplane needs 3 people to operate
- initially 110 planes ⟶ 330 crew members
- 1 crew member, not operating planes, can train 19 new crew members in 1 quarter
- crew operating in 1 quarter get leave in the next
- 20% get lost one the way back
- cost: new plane : 200 MU
  op./idle crew member: 7 MU
  new — " — : 10 MU
  resting — " — : 5 MU

- VARIABLES: p_i : # of new planes built in quarter i
  $(i = 1, \dots, 4)$ $\bar{p}_i$: # of idle planes — " —
  m_i : # of new crew members — " —
  $\bar{m}_i$: # of idle — " —

$\tilde{m}_i$: # of resting ——— " ———

(c_i : # of cargo units to be transported in quarter i)

- OBJECTIVE:

minimize $200 \sum_{i=1}^{4} p_i + 7 \sum_{i=1}^{4} \bar{m}_i + 10 \sum_{i=1}^{4} m_i + 5 \sum_{i=1}^{4} \tilde{m}_i$

- CONSTRAINTS:

$c_1 = 100$
$c_2 = 150$
$c_3 = 150$
$c_4 = 200$

(cargo that has to be transported)

$$c_1 + \bar{p}_1 = 110$$

$$c_2 + \bar{p}_2 = p_1 + \bar{p}_1 + 0.8\, c_1$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$3 \quad 3 \quad 2 \quad 2 \quad 2$$

$$4 \quad 4 \quad 3 \quad 3 \quad 3$$

(planes available/needed)

$$3c_1 + \bar{m}_1 + \tfrac{1}{20} m_1 = 330$$

$$\tilde{m}_1 = 0$$

$$3c_2 + \bar{m}_2 + \tfrac{1}{20} m_2 = m_1 + \bar{m}_1$$

$$\tilde{m}_2 = 0.8 \cdot 3 \cdot c_1 = 2.4\, c_1$$

$$3c_3 + \bar{m}_3 + \tfrac{1}{20} m_3 = m_2 + \bar{m}_2 + \tilde{m}_2$$

$$\vdots \qquad \vdots \qquad \vdots$$

(crew available/needed)

- LP: (matrix)

$A =$

c_1	c_2	c_3	c_4	$\bar{p}_1$	$\bar{p}_2$	$\bar{p}_3$	$\bar{p}_4$	p_1	p_2	p_3	p_4	$\bar{m}_1$	$\bar{m}_2$	$\bar{m}_3$	$\bar{m}_4$	m_1	m_2	m_3	m_4	$\tilde{m}_1$	$\tilde{m}_2$	$\tilde{m}_3$	$\tilde{m}_4$
1																				1			
	1																						
		1																					
			1																				
1				1				-1															
-0.8	1			-1	1				-1														
	-0.8	1			-1	1				-1													
		-0.8	1			-1	1				-1												
3												1				0.05							
	3											-1	1			-1	0.05						
		3											-1	1			-1	0.05		-1			
			3											-1	1			-1	0.05		-1		
2.4																				-1			
	2.4																				-1		
		2.4																				-1	

(right hand side)

$$b = \begin{pmatrix} 100 \\ 150 \\ 150 \\ 200 \\ \hline 110 \\ 0 \\ 0 \\ 0 \\ \hline 330 \\ 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$A \cdot x = b \quad , x \geq 0$$

↑

feasible set