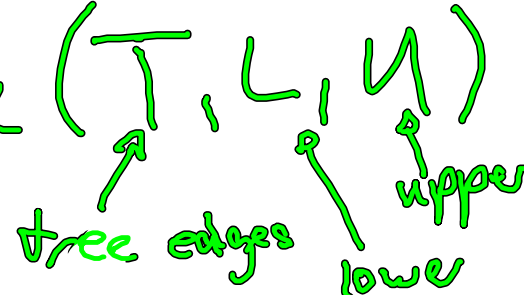


Maintain tree structure (T, L, U)

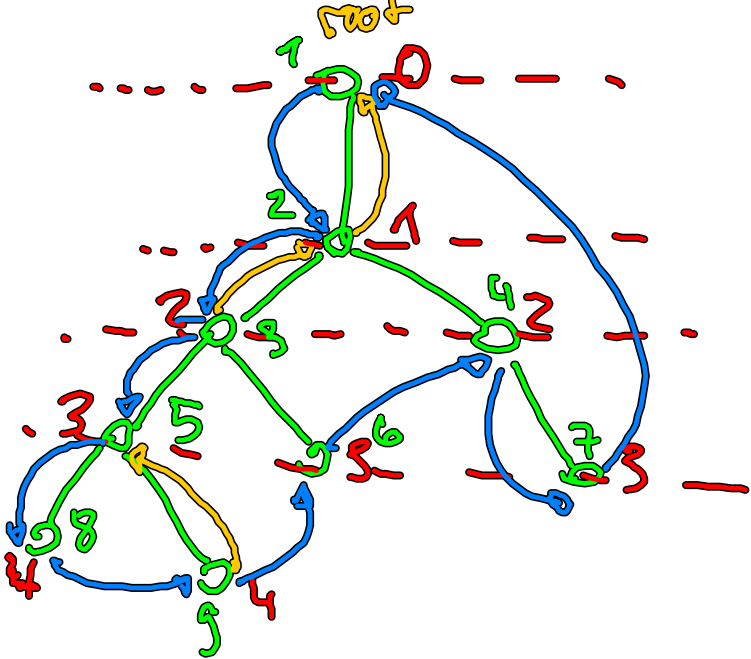


tree edges lower upper

- ① identify a cycle
↳ pivot cycle
 - ② compute and update
x flow and p dual values
 - ③ calculate θ and determine
the leaving arc
-

Saving the tree by indices at the nodes.

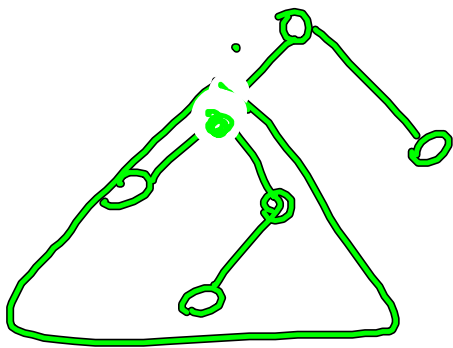
pred
depth
thread



depth first search

i	1	2	3	4
$pred(i)$	-	1	2	2
$depth(i)$	0	1	2	2
$thread(i)$	2	3	5	7

!



We can visit all nodes of a subtree without visiting the others by following the thread index.

Given the tree structure
compute the dual node potentials

begin

$p(1) := 0;$

$j := thread(1);$

while $j \neq 1$ do

```

begin
  i := pred(j);
  IF (i, j) ∈ T THEN
    p(j) := p(i) - cij;
  ← IF (j, i) ∈ T THEN
    p(j) := p(i) + cij;
  j := thread(j);
END;

```

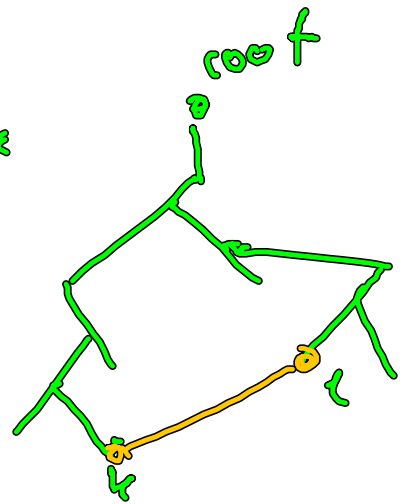
END;

Given the entering edge (k, l)
 We want to identify the pivot cycle

```

begin
  i := k;
  j := l;
  while i ≠ j do
    begin
      if depth(i) > depth(j) then
        i := pred(i);
      else if depth(j) > depth(i) then
        j := pred(j);
      else
        i := pred(i);
        j := pred(j);
      end;
    end;
  end;
end;

```

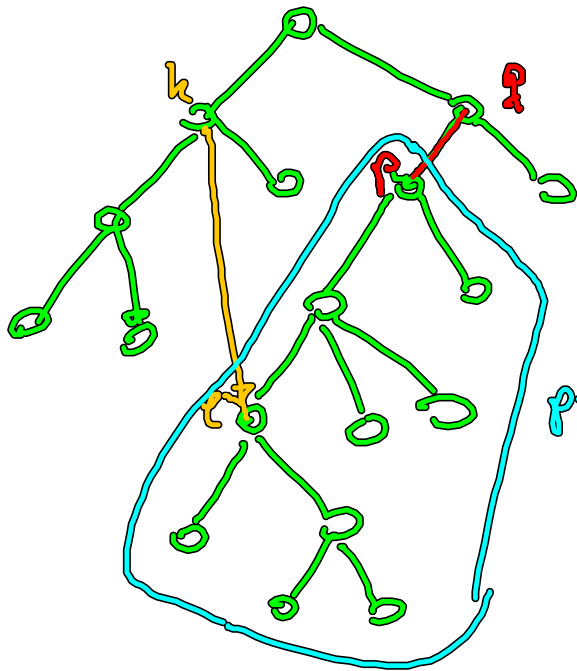


We use this method twice with two different side effects:

FIRST: Compute θ and leaving edge

SECOND: Update x flow values

FIRST update the node potentials



(p, q) leaving arc
 (k, l) entering arc
 $p(i) - p(j) = c_{ij} \quad \forall (i, j) \in E$

p -values will change

$$p(l)_{\text{new}} = p(k) - c_{kl}$$

$$\begin{aligned} p(l)_{\text{new}} - p(l)_{\text{old}} &= p(k) - c_{kl} - p(l)_{\text{old}} \\ &= -(c_{kl} - p(k) + p(l)_{\text{old}}) \\ &= -\bar{c}_{kl} \end{aligned}$$

IF l lies in the blue area, all p -value decrease by \bar{c}_{kl} .

IF k lies in the blue area, all p -value increase by \bar{c}_{kl} .

begin

$y := p;$

IF l lies in blue area THEN $change := -\bar{c}_{kl};$

IF k lies " THEN $change := \bar{c}_{kj};$

$p(y) := p(y) + change;$

$y := thread(y);$

while $depth(y) > depth(p)$ do

begin

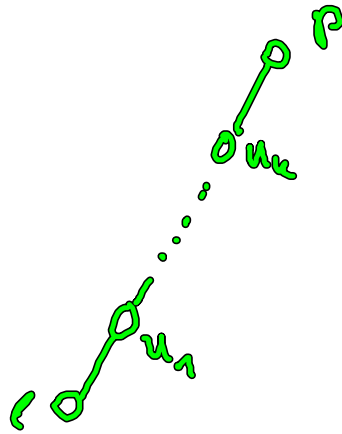
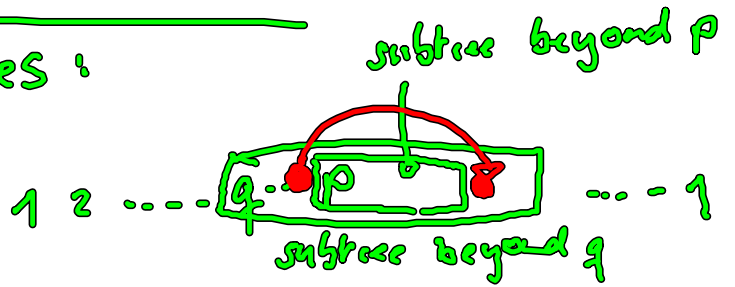
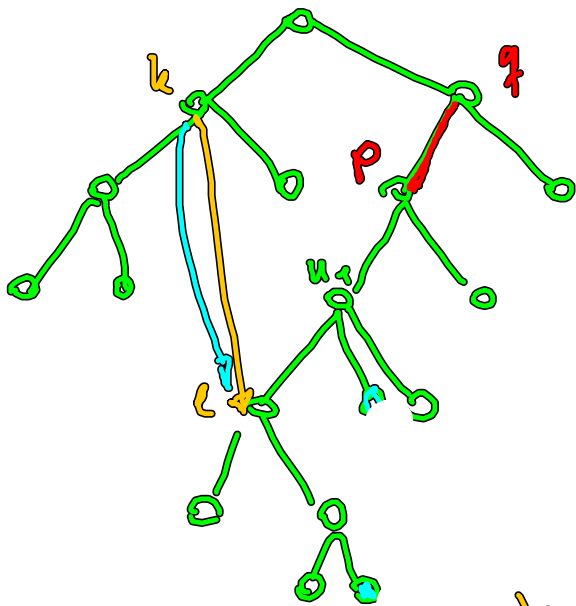
$p(y) := p(y) + change;$

$y := thread(y);$

end;

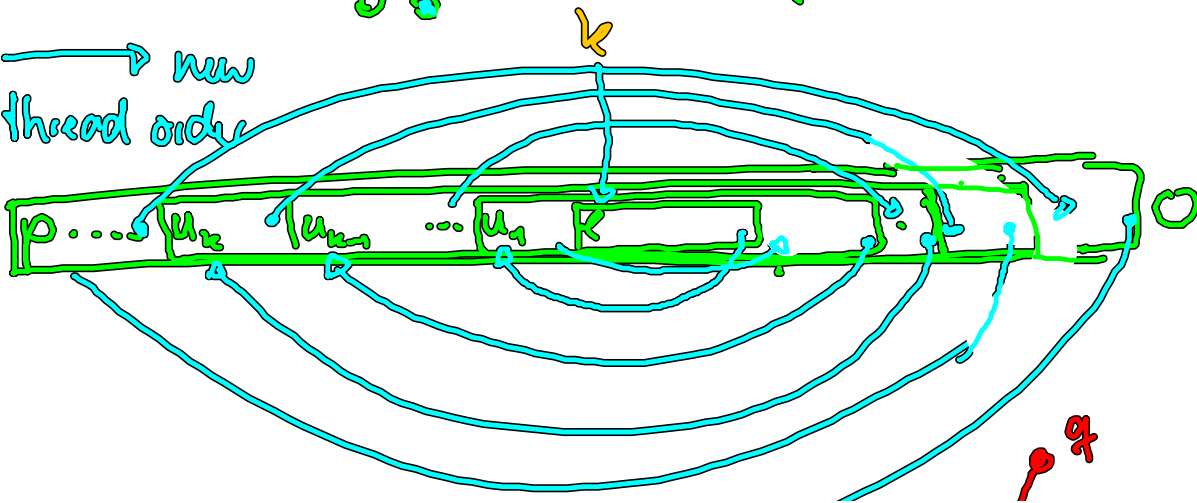
end;

Update the thread indices:

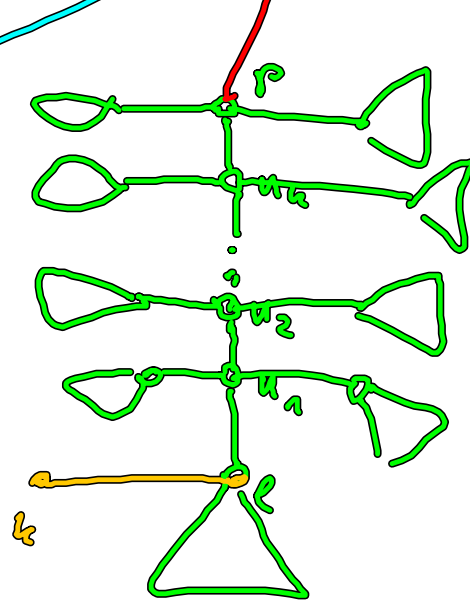


u_1, \dots, u_n are the inner nodes of the l - p -Path in tree T .

new thread order



$x = \text{thread}(k)$

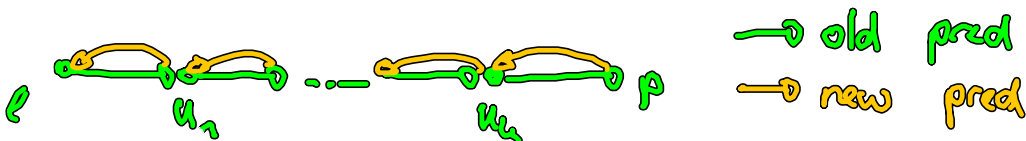


- 1) store $k, e, p, \text{thread}(k)$
- 2) identify (by pred-indices) and store u_1, \dots, u_k
- 3) Walk over subtree beyond p using the old thread-values and update them

thread-indices are updated correctly.

pred-indices are easy to update:

One has just to invert the $e-p$ -path



Do this as a side effect when identifying u_1, \dots, u_k .

pred-indices are updated correctly

depth-indices:

begin

$y := l;$

$\text{depth}(y) := \text{depth}(k) + 1;$

$y := \text{thread}(y);$

while $y \neq x$ do

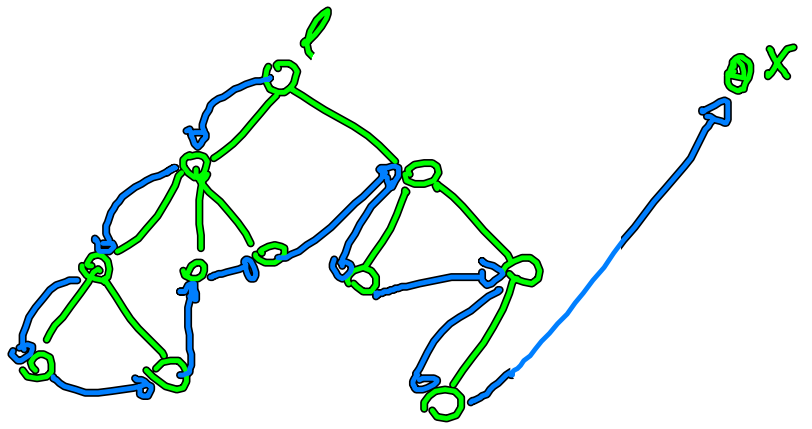
begin

$\text{depth}(y) := \text{depth}(\text{pred}(y)) + 1;$

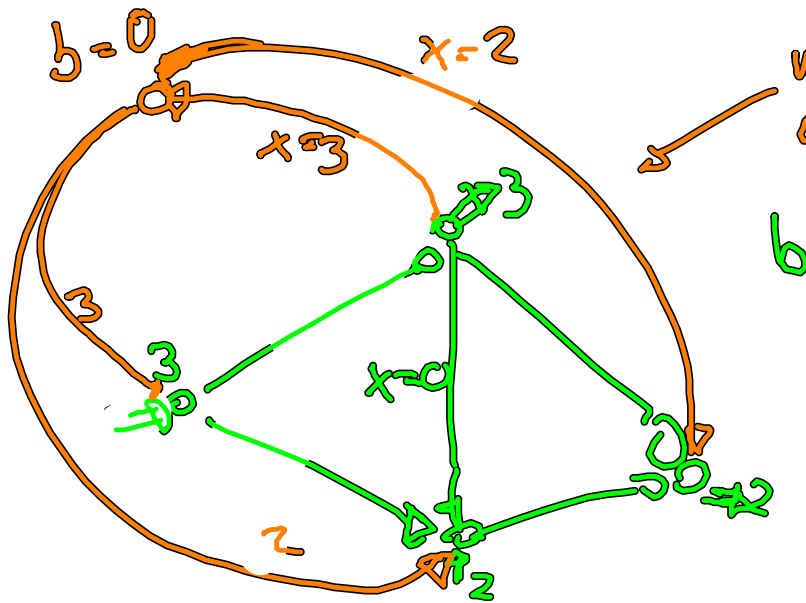
$y := \text{thread}(y);$

end;

end;



Depth-indices are updated correctly.



type capacities = ∞ (=M)
 cost = ∞ (=M = $\sum_{i,j \geq 0} c_{ij} + 1$)

Apply network simplex .

If one artificial arc is used in optimal solution the original problem is infeasible.