

Proof:

① suppose that rows $1, \dots, m$ are lex. positive. Suppose that x_j enters the basis and $x_{B(l)}$ exits the basis. By construction $u_e > 0$ ($u := B^{-1}A_j$) and

$$\frac{\text{l-th row}}{u_e} < \frac{\text{i-th row}}{u_i} \quad \text{for } i \neq l \text{ with } u_i > 0$$

Notice that

$$(\text{new l-th row}) = \frac{1}{u_e} \cdot (\text{old l-th row}) >^L 0$$

and for $i \neq l$

$$(\text{new i-th row}) = (\text{old i-th row}) - \frac{u_i}{u_e} (\text{old l-th row})$$

Case 1: $u_i \leq 0 \Rightarrow -\frac{u_i}{u_e} \geq 0 \Rightarrow$ new i-th row is the sum of a lexicogr. positive row and a lex. pos. or zero row \checkmark

Case 2: $u_i > 0 \Rightarrow \frac{1}{u_i} \cdot (\text{old i-th row}) >^L \frac{1}{u_e} (\text{old l-th row})$
 $\Rightarrow \text{old i-th row} - \frac{u_i}{u_e} \cdot (\text{old l-th row}) >^L 0$

② $\bar{c}_j < 0$ and

$$(\text{new 0-th row}) = (\text{old 0-th row}) - \underbrace{\frac{\bar{c}_j}{u_e}}_{> 0} \cdot \underbrace{(\text{old l-th row})}_{>^L 0} >^L 0$$

③ The zeroth row is uniquely determined by the current basis. As a consequence of ② the same basis cannot occur more than once. \square