

Proof:

① suppose that rows $1, \dots, m$ are lex. positive. Suppose that x_j enters the basis and $x_{B(\ell)}$ exits the basis. By construction $u_\ell > 0$ ($u_i := B^{-1} \cdot A_j$) and

$$\frac{\ell\text{-th row}}{u_\ell} < \frac{i\text{-th row}}{u_i} \quad \text{for } i \neq \ell \text{ with } u_i > 0$$

Notice that

$$(\text{new } \ell\text{-th row}) = \frac{1}{u_\ell} \cdot (\text{old } \ell\text{-th row}) > \frac{1}{u_\ell} \cdot 0$$

and for $i \neq \ell$

$$(\text{new } i\text{-th row}) = (\text{old } i\text{-th row}) - \frac{u_i}{u_\ell} (\text{old } \ell\text{-th row})$$

Case 1: $u_i \leq 0 \Rightarrow -\frac{u_i}{u_\ell} \geq 0 \Rightarrow$ new i -th row is the sum of a lexicogr. positive row and a lex. pos. or zero row \checkmark

Case 2: $u_i > 0 \Rightarrow \frac{1}{u_i} \cdot (\text{old } i\text{-th row}) > \frac{1}{u_\ell} \cdot (\text{old } \ell\text{-th row})$
 $\Rightarrow \text{old } i\text{-th row} - \frac{u_i}{u_\ell} \cdot (\text{old } \ell\text{-th row}) > \frac{1}{u_\ell} \cdot (\text{old } \ell\text{-th row}) > 0$

② $\bar{z}_j < 0$ and

$$(\text{new } 0\text{-th row}) = (\text{old } 0\text{-th row}) - \underbrace{\frac{\bar{z}_j}{u_\ell}}_{> 0} \cdot \underbrace{(\text{old } \ell\text{-th row})}_{> 0} > \frac{1}{u_\ell} \cdot (\text{old } \ell\text{-th row}) > 0$$

③ The zeroth row is uniquely determined by the current basis. As a consequence of ② the same basis cannot occur more than once. \square