Traffic Flow Optimization under Fairness Constraints

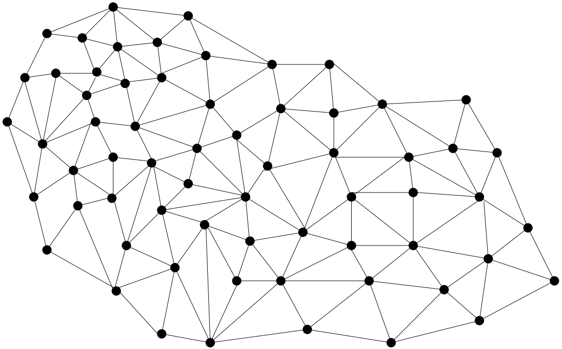
Combinatorial Optimization & Graph Algorithms (COGA)

Technische Universität Berlin

Outline

Route Guidance: Introduction

- given: sources s_k, targets t_k, demand rates d_k for traffic demands in a road network



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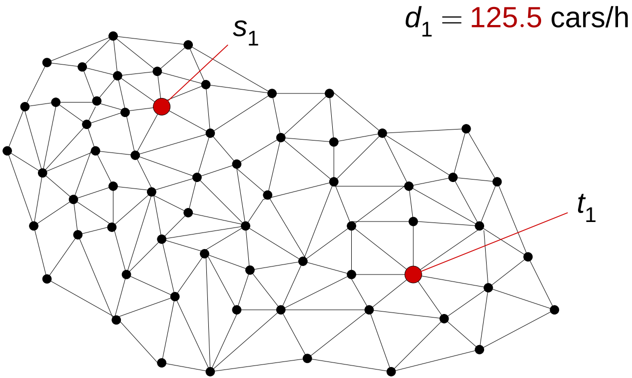

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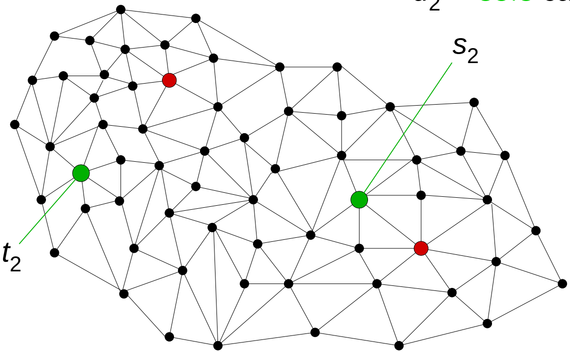

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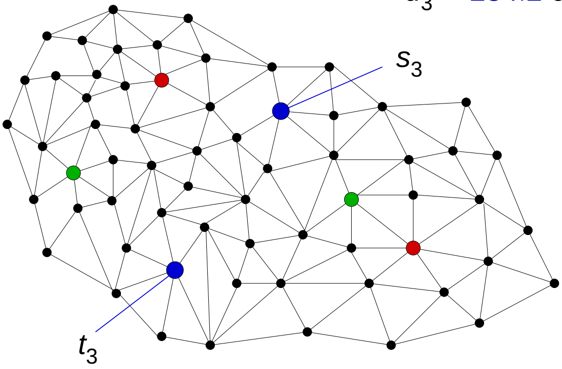

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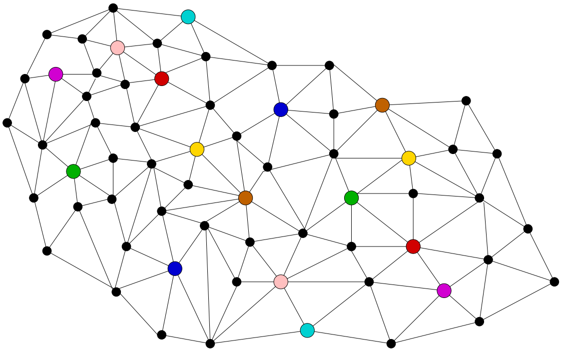

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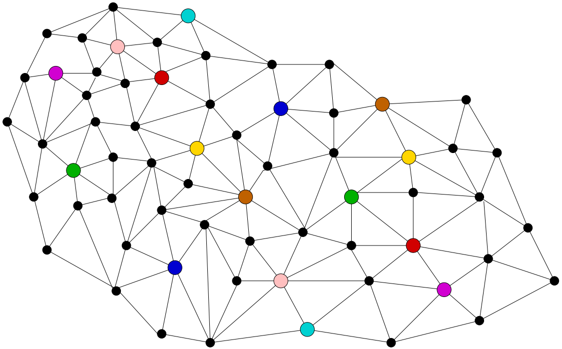

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Route Guidance: State of Technology

Route Guidance Systems...

... play an increasingly important role in today's traffic:

- in-car navigation systems
- urban road pricing schemes / centralized traffic routing

Today's systems use static data only:

- average travel times on road links
- locations / times of typical rush hour congestions
- locations of work zones

⇒ routes computed by static shortest path calculations

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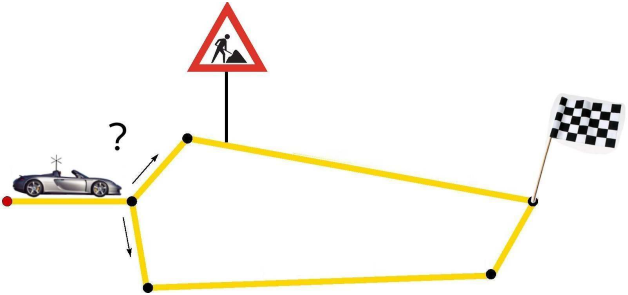
Results of Widespread Static Route Guidance

Travelers with the same origin and destination receive the same route suggestions:



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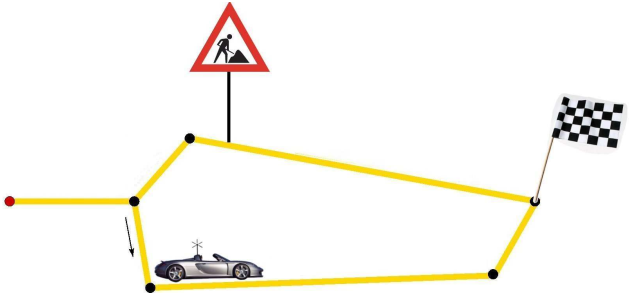

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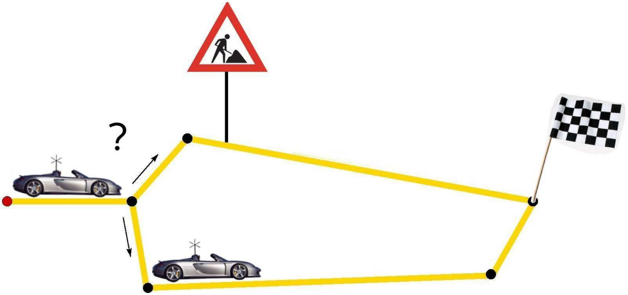

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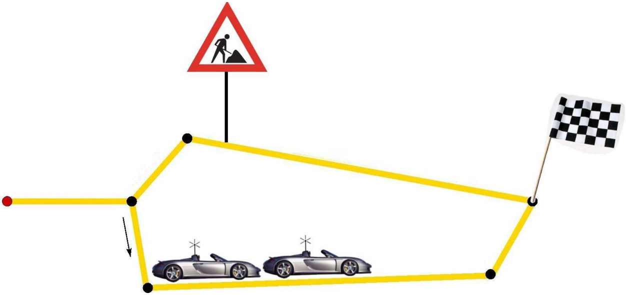

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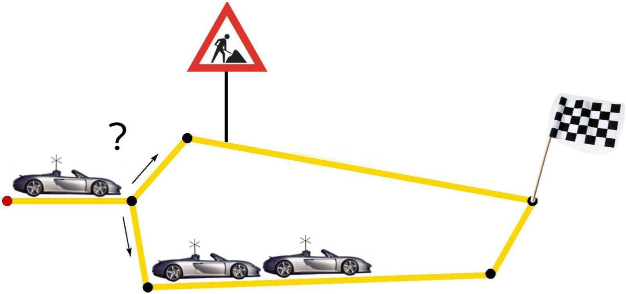

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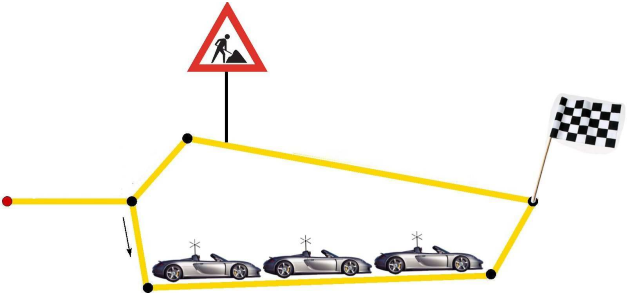

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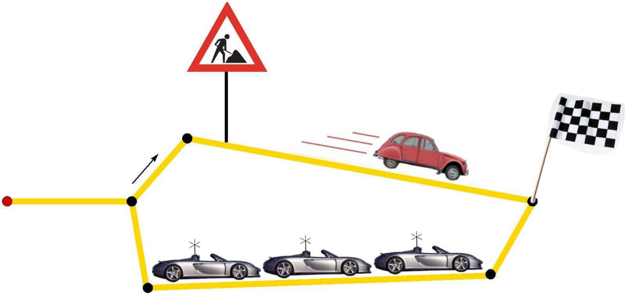

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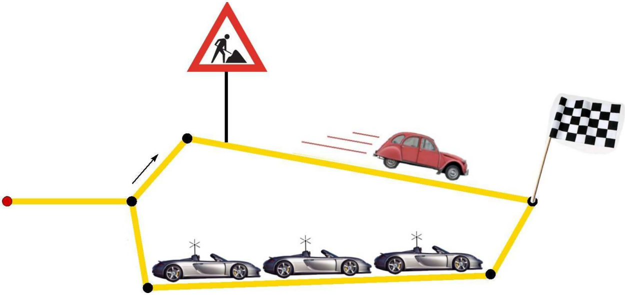

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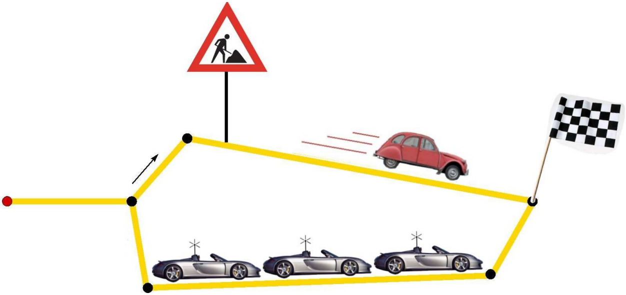

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The Need for Intelligent Traffic Routing

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System Optimum

Sum of all travel times is minimal.

Problems (e.g. [Mahmassani and Peeta 1993]):

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No user can improve his travel time by individually changing his route.

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Result:

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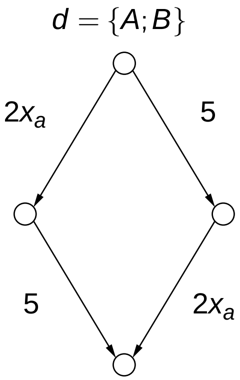

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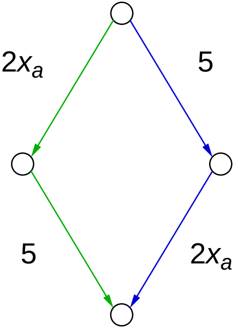
$d = \{A; B\}$



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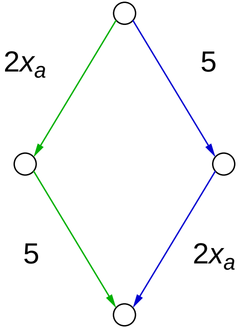

$\tau(A) = \tau(B) = 7$

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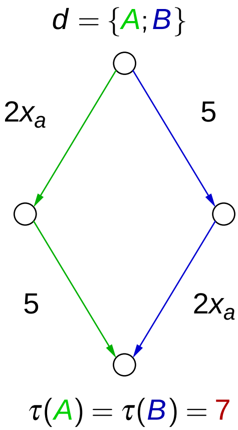

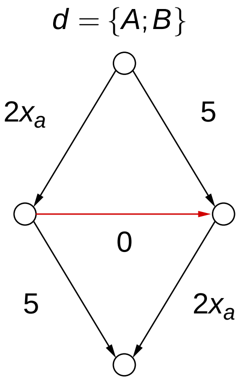

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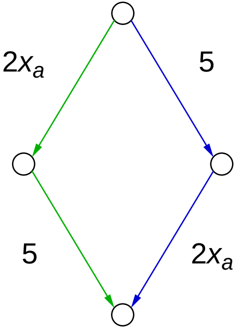

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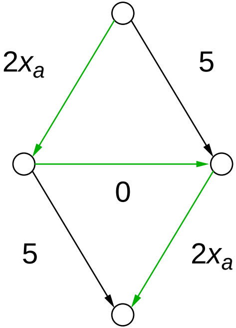

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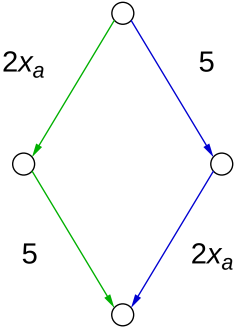

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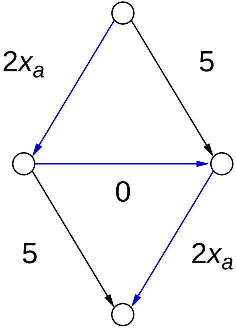

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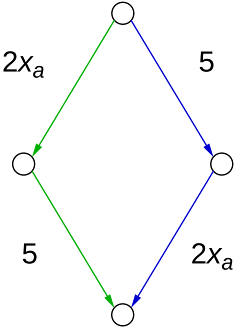

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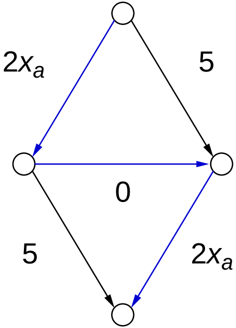

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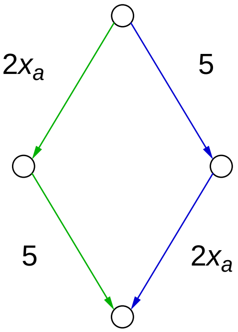

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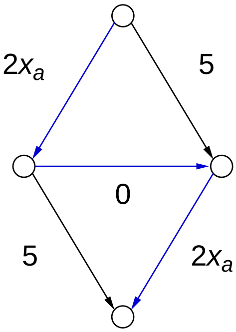

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System Optimum with Fairness Constraints

Idea [Jahn, Möhring, Schulz, Stier-Moses 2005]

Minimize sum of all travel times, but restrict usage of paths drivers would not accept:

- $\tau_p :=$ travel time on path p in UE
- $T_k :=$ travel time on paths chosen by commodity k in UE

$\Rightarrow$ only use paths p with

$$\tau_p \leq \varphi \cdot T_k$$

- suggestion: $\varphi = 1.02$

$\Rightarrow$ drivers are suggested paths which they think are fair!

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Properties of the Constrained System Optimum

Results [Jahn, Möhring, Schulz, Stier-Moses 2005]

With appropriate φ, τ, solutions to CSO yield

- a lot more fairness than System Optimum
  - travel time of 99% of all users at most 30% higher than on fastest route.
  - in SO: 50%
- much better system performance than User Equilibrium
  - total travel time only $\frac{1}{3}$ as far away from SO as UE
- better routes for most drivers
  - 75% spend less travel time than in UE
  - only 0.4% spend 10% more (SO: 5%)

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The CSO Problem

min-cost multi-commodity flow problem with convex objective function and path constraints:

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& \sum_{p \in P_k : a \in p} x_p = z_a^k & a \in A \\
& \sum_{p \in P_k} x_p = d_k & k \in K \\
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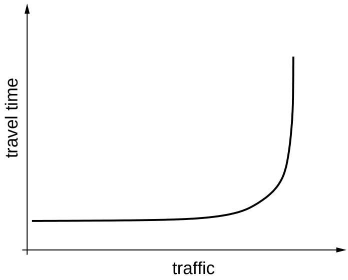

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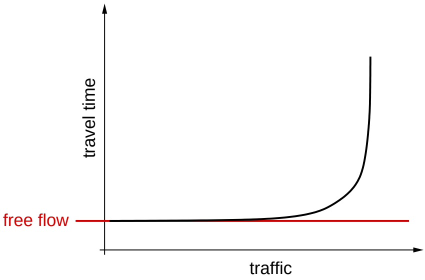

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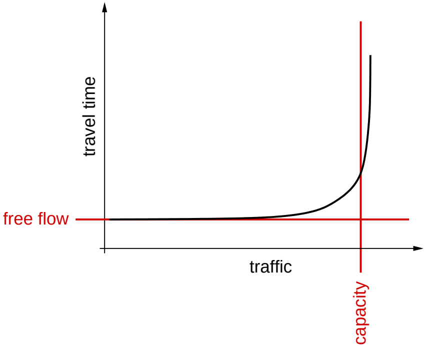

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  - $\Rightarrow$ cannot deal with variables x_p explicitly

Previous work [Jahn, Möhring, Schulz, Stier-Moses 2004]:

- solve CSO by variant of Frank-Wolfe convex combinations algorithm and constrained shortest path calculations

$\Rightarrow$ runtime acceptable: instances with a few thousand nodes / arcs / commodities take some minutes

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A Different Approach

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- define appropriate Lagrangian relaxation
- use cutting plane method to solve dual problem

- similar approach successfully applied to other multi-commodity flow problems [Babonneau and Vial 2005]

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- define appropriate Lagrangian relaxation
- use cutting plane method to solve dual problem

- similar approach successfully applied to other multi-commodity flow problems [Babonneau and Vial 2005]

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Outline

Lagrangian Relaxation for CSO

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\begin{aligned}
&\textit{Minimize} && L(x,u) := \sum_{a\in A} l_a(x_a)x_a && \\
&\textit{subject to} && \sum_{p\in P_k : a\in p} x_p = z_a^k && a \in A \\
& && \sum_{p\in P_k} x_p = d_k && k \in K \\
& && \tau_p \le \varphi T_k && p \in P^k : x_p > 0;\ k \in K \\
& && x_p \ge 0 && p \in P \\
& && \sum_{k\in K} z_a^k = x_a && a \in A
\end{aligned}
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Lagrangian Relaxation for CSO

- drop constraints coupling total and commodity flows

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Lagrangian Relaxation for CSO

- add penalty terms with multipliers u_j to objective

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Lagrangian Relaxation for CSO

- add penalty terms with multipliers u_j to objective

$$\text{Minimize} \quad L(x,u) := \sum_{a\in A} l_a(x_a)x_a + \sum_{a\in A}\left(u_a \cdot \left(\sum_{k\in K} z_a^k - x_a\right)\right)$$

$$\text{subject to} \quad \sum_{p\in P_k : a\in p} x_p = z_a^k \qquad a\in A$$

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$$x_p \ge 0 \qquad p\in P$$

Lagrangian Relaxation for CSO

- remaining constraints resemble those of $|K|$ constrained shortest path problems in z_a^k

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Lagrangian Relaxation for CSO

- Lagrangian separable in x and z?

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Lagrangian Relaxation for CSO

- Lagrangian separable in x and z?

$$\begin{aligned}
&\textit{Minimize} && L(x,u) := \overbrace{\sum_{a\in A}(l_a(x_a)-u_a)\cdot x_a}^{L_1(x,u)} + \overbrace{\sum_{k\in K}\sum_{a\in A} u_a\cdot z_a^k}^{L_2(z,u)} \\
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\end{aligned}$$

Lagrangian Relaxation for CSO

⇝ Yes!

$$\textit{Minimize} \quad L(x,u) := \overbrace{\sum_{a\in A} (l_a(x_a) - u_a)\cdot x_a}^{L_1(x,u)} + \overbrace{\sum_{k\in K}\sum_{a\in A} u_a \cdot z_a^k}^{L_2(z,u)}$$

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Lagrangian Relaxation for CSO

- easier problem: analytical minimization in x...

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Lagrangian Relaxation for CSO

- ...and $|K|$ constrained shortest path problems in z^k

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Lagrangian Relaxation for CSO

- up next: dual problem (maximize this minimum over u)

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Outline

Analytic Center Cutting Plane Method

- approximation scheme for maximization of a concave function over a convex set
- implementation by Babonneau, Vial et. al. at LogiLab, University of Geneva

Two components:

- query point generator
  - manages a localization set containing all optimal points
  - selects query points which are tried for optimality
- oracle
  - generates cutting planes to further bound the localization set
  - problem dependent!

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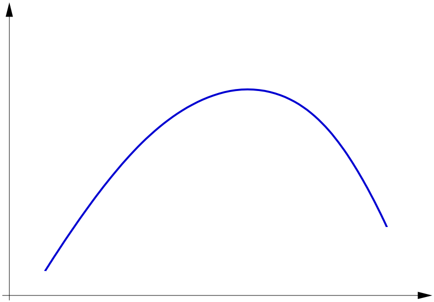
Oracle for CSO

- evaluate objective function ⇝ CSP calculations
- calculate subgradient at query point ⇝ easy

⇒ subgradients and best objective value define cutting planes bounding the localization set



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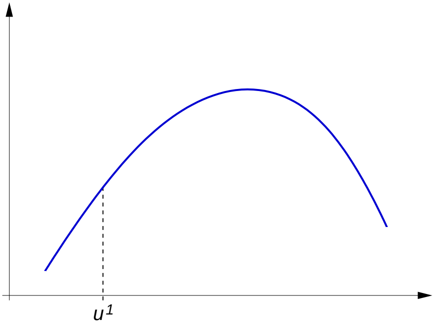

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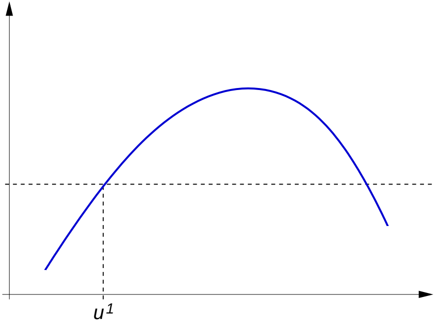

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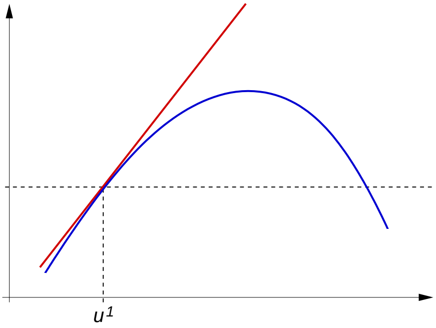

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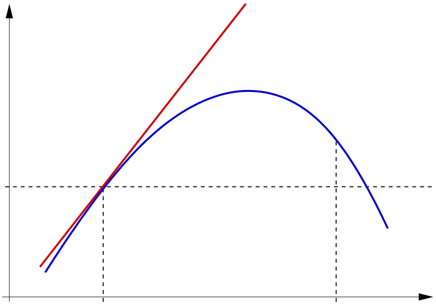

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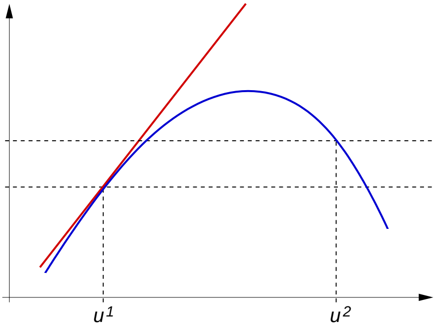

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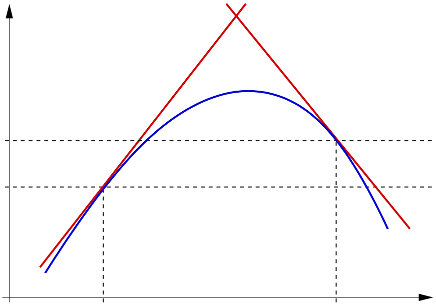

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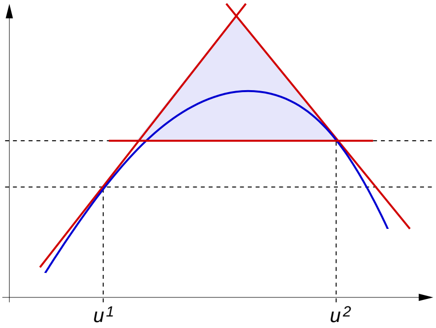
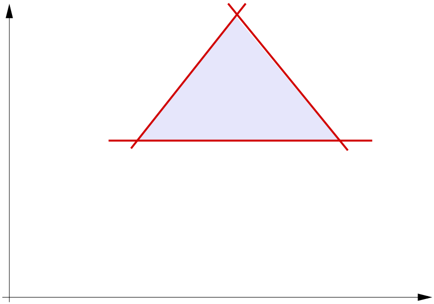


Query Points

- analytic center: maximum distances from cutting planes
  - calculation by damped Newton method
- *u*-component is next query point



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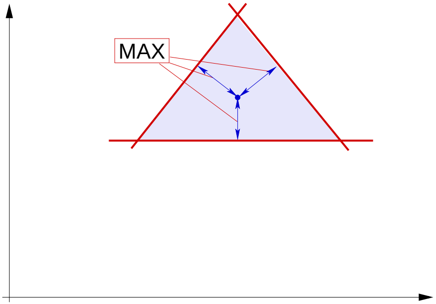

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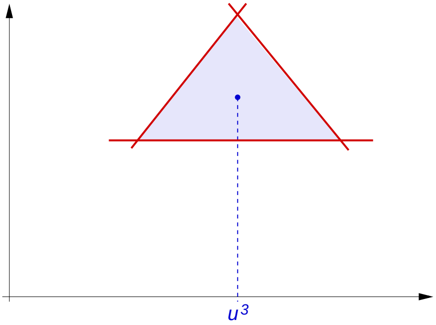

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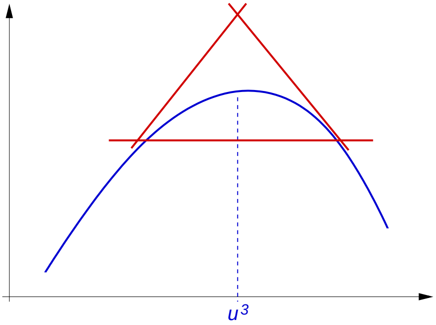
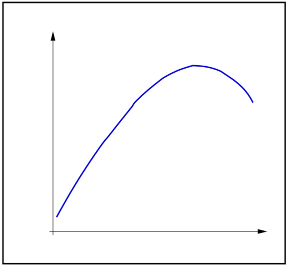
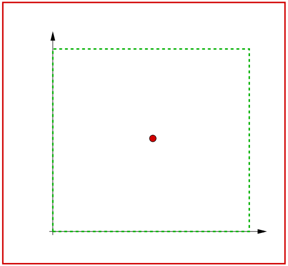


Illustration of an ACCPM Run


oracle


query point generator

- localization set artificially bounded $\Rightarrow$ compact

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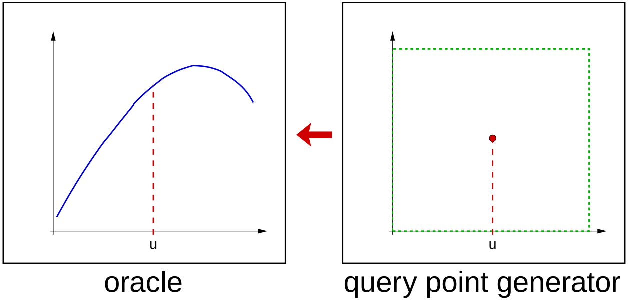

In each iteration, a query point is sent to the oracle,...

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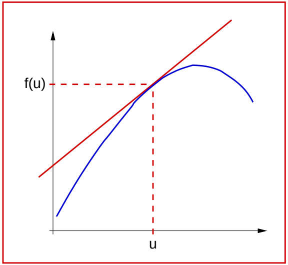

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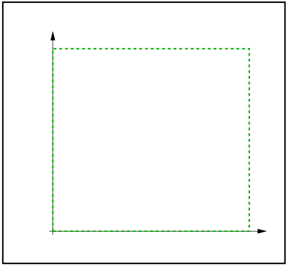
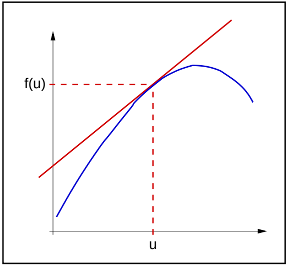


query point generator

... the value and subgradient of θ are calculated...

Illustration of an ACCPM Run



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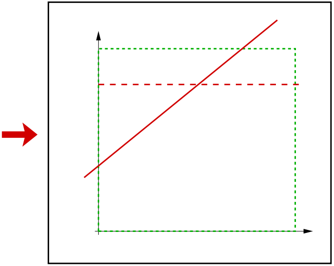

query point generator

... which define cutting planes...

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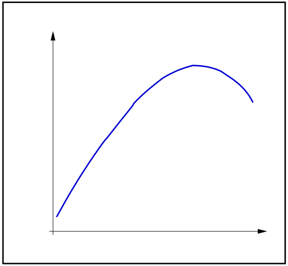

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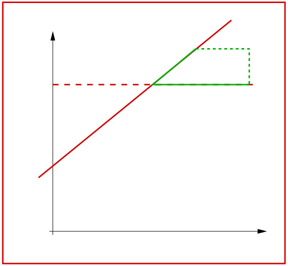

query point generator

... to further bound the localization set.

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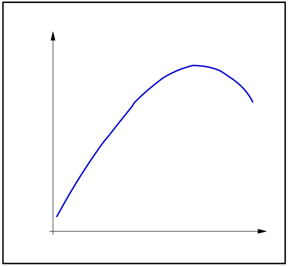

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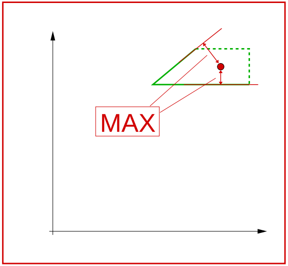

query point generator

Then, the proximal analytic center is calculated...

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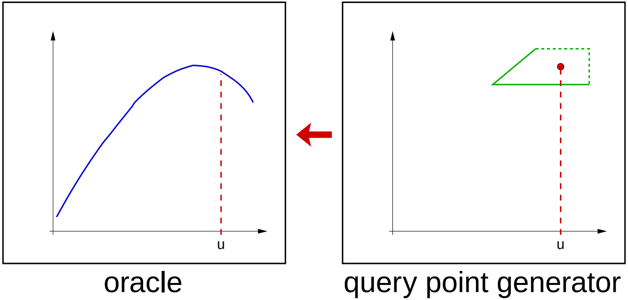

... which defines the next query point.

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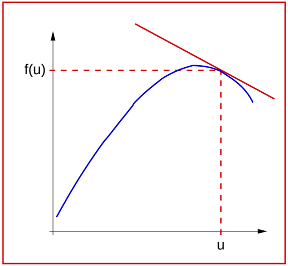

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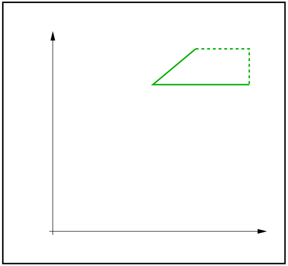

query point generator

Process is repeated...

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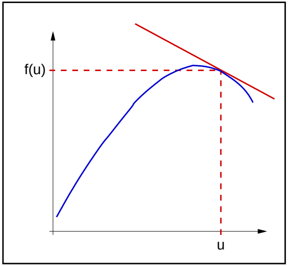

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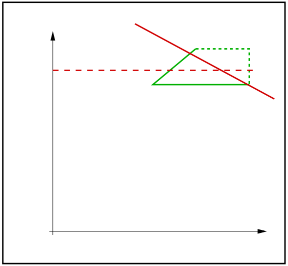

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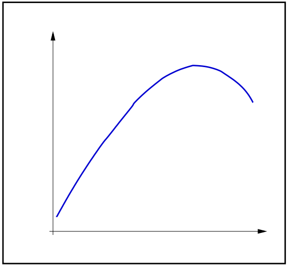

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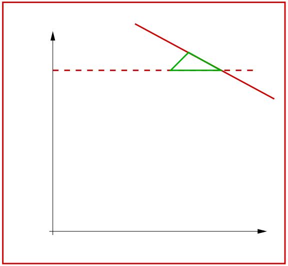

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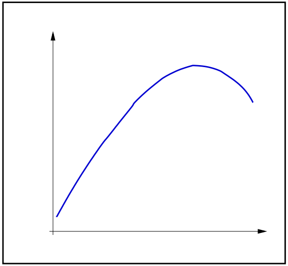

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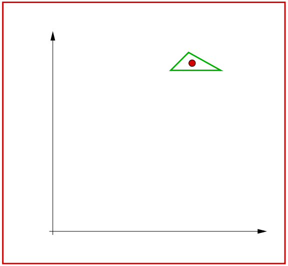

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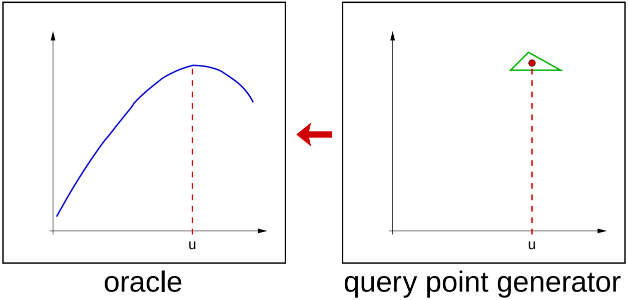

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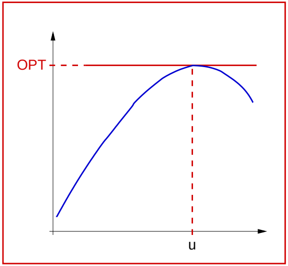

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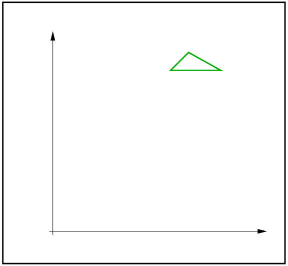

query point generator

... until desired precision is achieved.

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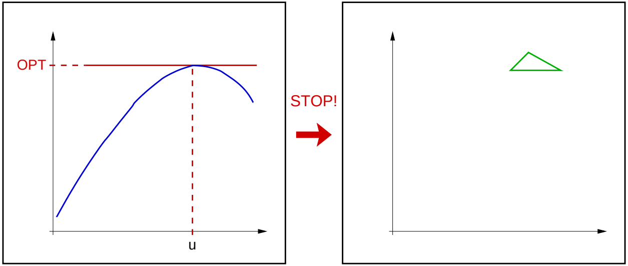

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There is much more inside ACCPM

Accelerating convergence:

- sophisticated parameters for dynamic weighting of cuts
- cut elimination techniques
- rules for updating the proximal reference point

Problem specific functionalities:

- multiple cuts per iteration
- active set strategies

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...

Outline

The Algorithm

Algorithm

- define Lagrangian Relaxation of CSO
- use proximal ACCPM to solve Lagrangian dual problem
  - oracle: solve CSP problems
    $\Rightarrow$ primal lower bound
  - query point generator: damped Newton method
- primal solution / upper bound through heuristic: convex combination of paths from oracle
- stop when desired precision guaranteed

- different variants of labeling algorithm for CSP: basic, bidirectional, goal-oriented

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Test Instances

Name	$\|V\|$	$\|A\|$	$\|K\|$
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`Winnipeg`	1052	2836	4344
`Neukoelln`	1890	4040	3166
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- all but `Neukoelln` from Transportation Network Problems online database
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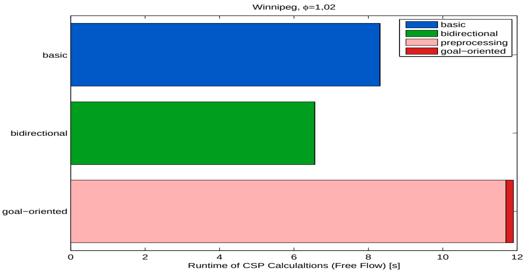

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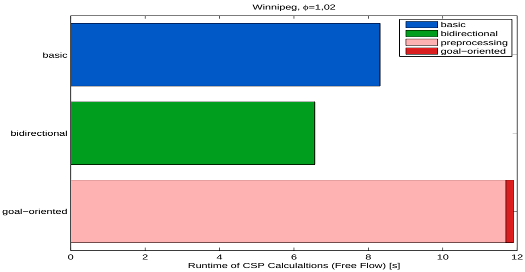

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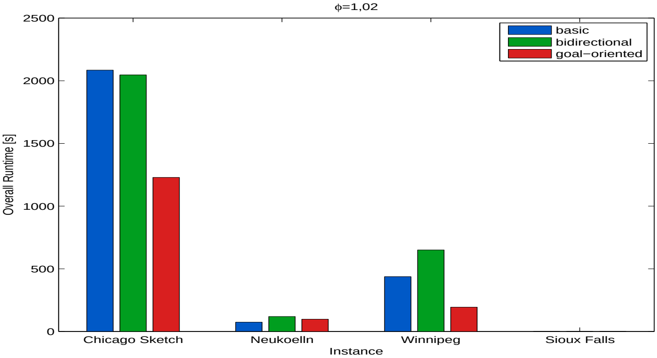
- total runtime: basic / bidirectional / goal-oriented



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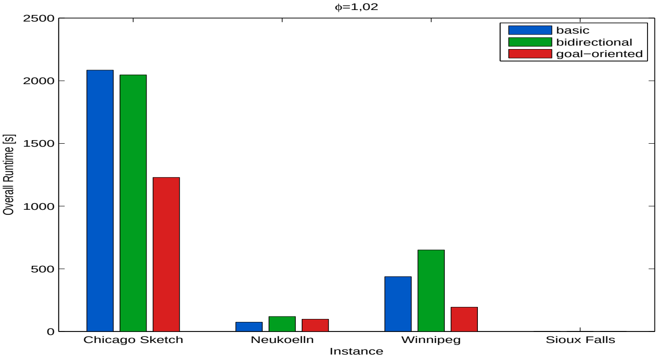

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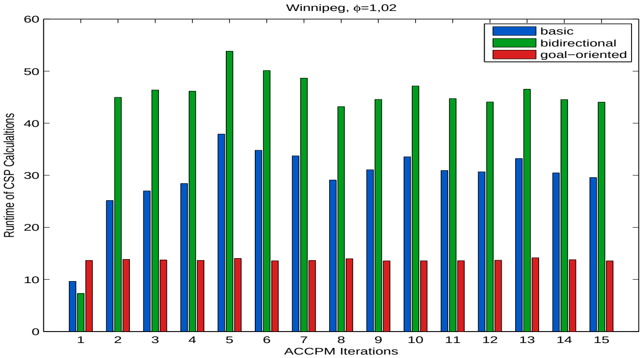
Effect of CSP Acceleration

- CSP-runtime over ACCPM iterations for Winnipeg



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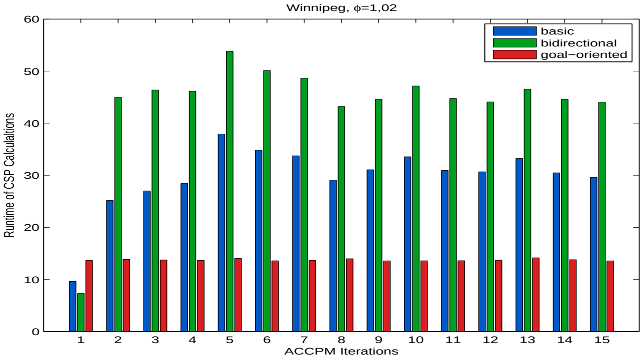

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Explanation

- edge length for CSP calculations: dual variables *u*
- as we approach optimum, *u* approaches CSO travel times

⇒ in congested networks, direct paths become unattractive

⇒ basic labeling algorithm is deflected from target

  ⇝ infeasible paths are explored first

- goal orientation dominates this effect

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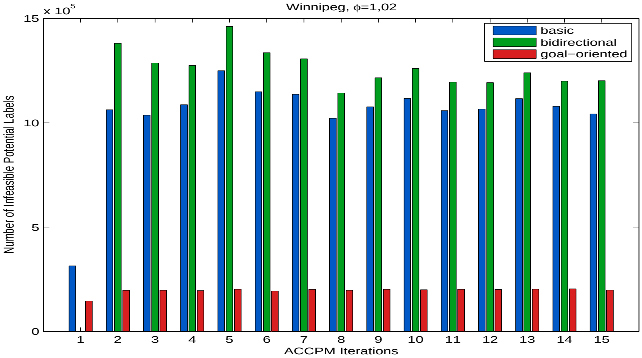
Exploration of Nodes on Infeasible Paths

- potential labels violating length bounds for Winnipeg



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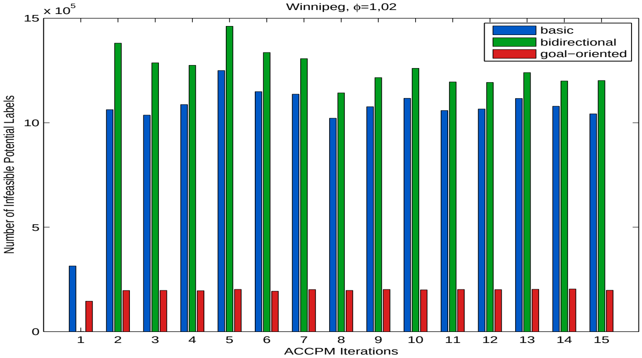

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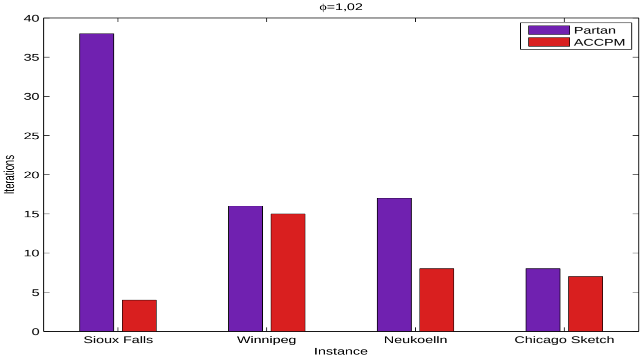
Comparison with Partan

- number of iterations compared with Partan



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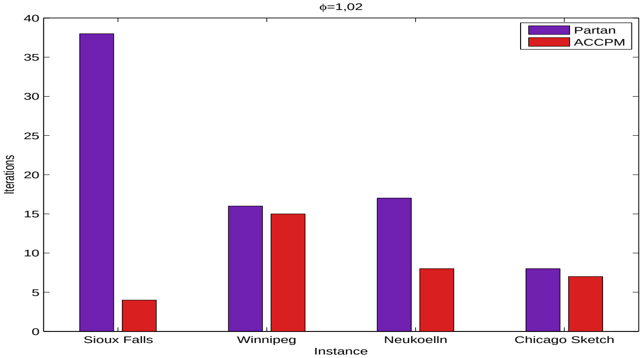

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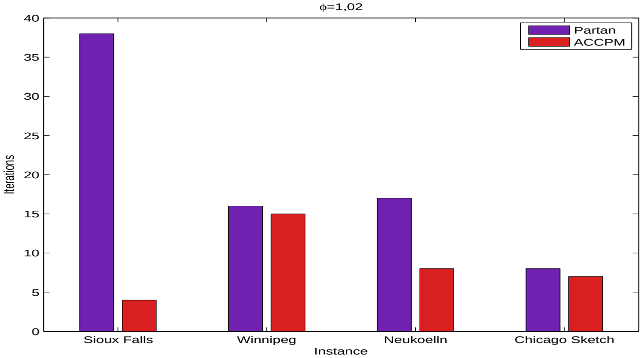

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- proximal-ACCPM can be used to solve a Lagrangian relaxation of CSO
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  $\approx \frac{1}{2}$ the runtime of Partan algorithm
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  $\leftrightarrow$ level of congestion
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