Topology

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Problem Set 11

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Exercise 70. **6 points**

Let $X = (I \times I)/\sim$ with $\sim$ induced by $(x, 0) \sim (x, 1)$ and $(0, y) \sim (1, 1 - y)$, the Klein bottle.

(a) Describe $\pi_1(X)$ with the help of Proposition 11.6.

(b) Consider the group of all homeomorphisms $\mathbb{R}^2 \to \mathbb{R}^2$, and let G be the subgroup generated by the two functions $f(x, y) = (x, y + 1)$ and $g(x, y) = (x + 1, 1 - y)$. Show that there is an epimorphism $\pi_1(X) \to G$.

Remark: In fact, $\pi_1(X) \cong G$.

Exercise 71. **4 points**

Find all homomorphisms from $\pi_1(S^3 \setminus K_{2,5})$ to the symmetric group S_3, and show that the knots $K_{2,3}$ and $K_{2,5}$ are not equivalent.

Exercise 72. **6 points**

Let $X = (S^1 \times I)/\sim$ with the equivalence relation

$$(x, y) \sim (x', y') \iff (y = y' \wedge (x = x' \vee (x = -x' \wedge y \in \{0, 1\}))).$$

Compute $\pi_1(X)$, and show that $\pi_1(X)$ is not abelian and not finite.

Exercise 73. **(Tutorial)**

Compute $\pi_1(S^2 \cup D^1)$.

Exercise 74. **(Tutorial)**

Show that every finite group is the fundamental group of some space.

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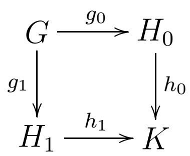
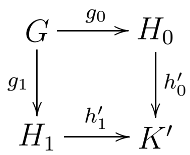
Exercise 75.

Proposition 11.10 (Universal property of push-out). Consider the following diagram of groups and homomorphisms.



This diagram is a push-out diagram if and only if it commutes and for every commuting diagram



there exists a unique homomorphism $k : K \to K'$ with $k \circ h_0 = h'_0$ and $k \circ h_1 = h'_1$, that is, the following diagram commutes: