Technische Universität Berlin
Fakultät II – Institut für Mathematik

Berlin Mathematical School
Wintersemester 2008/2009

Topology

Dr. Carsten Schultz

Axel Werner

Problem Set 12

(last of semester)

Deadline: 28 Jan 2009

Exercise 76. **4 points**

Let $\mathcal{S}$ be an abstract simplicial complex.

(a) Let $A \subset |\mathcal{S}|$ such that for every $\sigma \in \mathcal{S}$ the set $\chi_\sigma^{-1}[A]$ is finite. Show that A is closed in $|\mathcal{S}|$ and discrete.

(b) Let $K \subset |\mathcal{S}|$ be quasi-compact. Show that there is a finite subcomplex $\mathcal{T}$ of $\mathcal{S}$ such that $K \subset |\mathcal{T}|$.

Exercise 77. **6 points**

Consider the following simplicial complex on the set $\{1, \dots, 6\}$:

$$\begin{aligned} \mathcal{S} \;=\; & \left\{ \, S \subset \{1,\dots,6\} \mid |S| \le 2 \, \right\} \cup \\ & \left\{ \, \{1,2,4\}, \{1,2,6\}, \{1,3,4\}, \{1,3,5\}, \{1,5,6\}, \right. \\ & \quad \left. \{2,3,5\}, \{2,3,6\}, \{2,4,5\}, \{3,4,6\}, \{4,5,6\} \, \right\} \end{aligned}$$

Use Theorem 12.20 to compute the fundamental group of $|\mathcal{S}|$. Do you recognise the space $|\mathcal{S}|$?

Exercise 78. **6 points**

For $m, n \in \mathbb{N}$, the (m,n)-*chessboard complex* $\Delta_{m,n}$ is the abstract simplicial complex

$$\big\{ S \subset \{1,\dots,m\} \times \{1,\dots,n\} \;\big|\; (r,s),(r',s') \in S \implies (r,s) = (r',s') \vee (r \ne r' \wedge s \ne s') \big\}.$$

Compute the fundamental group of the chessboard complex $\Delta_{4,3}$.
Hint: You can either draw a figure of $|\Delta_{4,3}|$ and recognise it or use the method from the lectures to compute the fundamental group.

please turn over

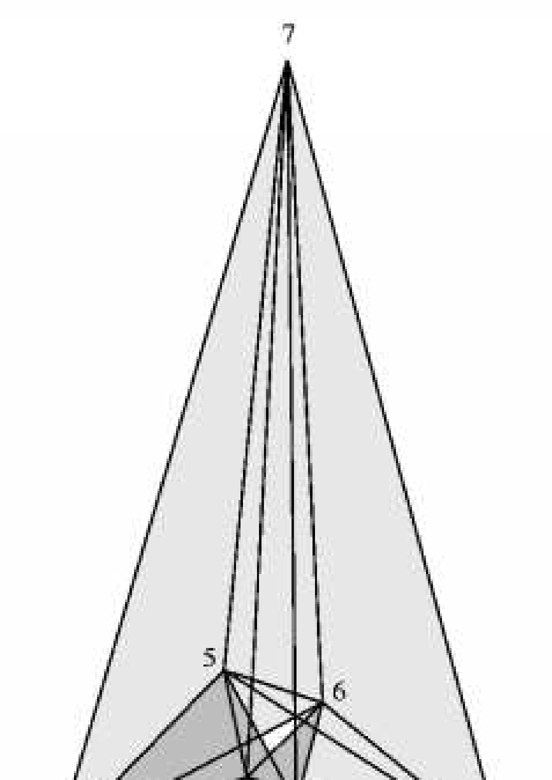
Exercise 79. **(Tutorial)**

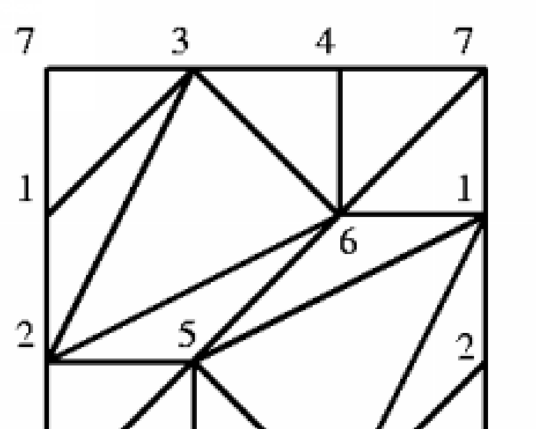
(a) Give a triangulation of the Klein bottle and use Theorem 12.20 to compute its fundamental group.

(b) Show that the two spaces in Exercises 70 and 72 are homeomorphic and give an isomorphism between the different presentations of the fundamental group you obtained in the exercises.

Exercise 80. **(Tutorial)**

Let $\mathcal{S}$ be an abstract simplicial complex, Y a space, and $(f_\sigma)_{\sigma \in \mathcal{S}}$ a system of continuous maps $f_\sigma : \Delta^\sigma \to Y$. Show that there is a continuous map $f : |\mathcal{S}| \to Y$ such that $f \circ \chi_\sigma = f_\sigma$ for all $\sigma \in \mathcal{F}$ if and only if $f_\tau = f_\sigma \circ i_\tau^\sigma$ for all $\tau \subset \sigma \in \mathcal{F}$.





http://www.eg-models.de/models/Classical_Models/2001.02.069/