Technische Universität Berlin
Fakultät II – Institut für Mathematik

Berlin Mathematical School
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Topology

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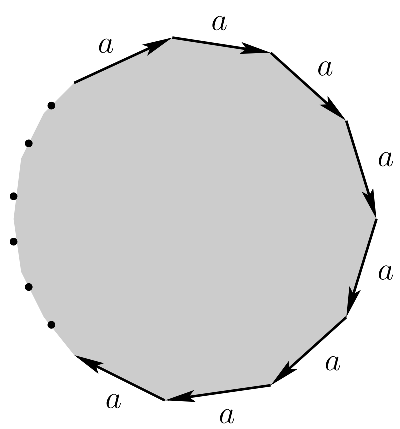
Problem Set 13

(*really* last of semester)

Deadline: 4 Feb 2009

❄ Exercise 81. 6[†] extra points

For $n \geq 2$ consider the space X_n, obtained from a (filled) n-gon, where boundary edges are identified as indicated in the sketch.



Compute the fundamental group of X_n using your favourite method.

❄ Exercise 82. 2 extra points

Compute the homology groups $H_0(X_n;\mathbb{Z})$ and $H_1(X_n;\mathbb{Z})$ of the space X_n from the previous exercise.

[†]3 points for the correct solution, plus 3 points for formal correctness and understandability—note that "formal correctness" does not necessarily mean that you have to give detailed formulas for each argument, but rather that mathematically nonsensical expressions are to be avoided!

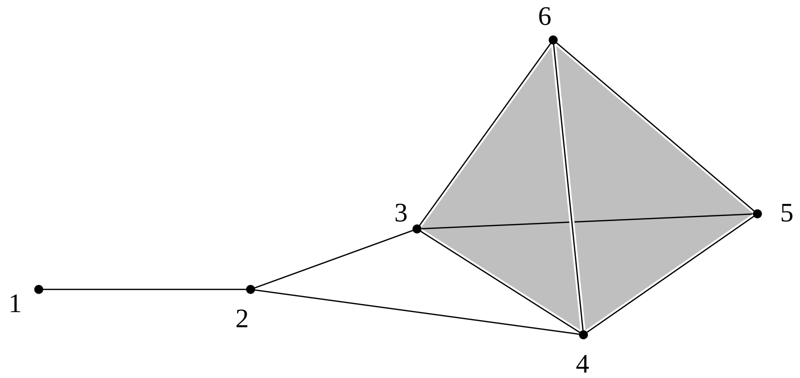
Exercise 83. **(Tutorial)**

Compute the homology of a triangulation of $S^1 \vee S^1$ with respect to $\mathbb{Z}$.

Exercise 84. **(Tutorial)**

Compute the homology $H_k(\mathcal{S}; \mathbb{Z})$ of the simplicial complex

$$\begin{aligned} \mathcal{S} \;=\; \big\{ \, & \emptyset,\ \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}, \\ & \{1,2\}, \{2,3\}, \{2,4\}, \{3,4\}, \{3,5\}, \{3,6\}, \{4,5\}, \{4,6\}, \{5,6\} \\ & \{3,4,5\}, \{3,4,6\}, \{3,5,6\}, \{4,5,6\} \ \big\} \end{aligned}$$



Exercise 85. **(Tutorial)**

Let $\mathcal{S}$ be a triangulation of the projective plane $\mathbb{R}P^2$. Compute the homology modules $H_k(\mathcal{S}; R)$ for $k = 0, 1, 2$ and $R = \mathbb{Z}, \mathbb{Q}, \mathbb{Z}_2$.