

Topology

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Repetition

Question 86.

- (a) What is a topological space? What defines open sets, closed sets and neighbourhoods? What is a (neighbourhood) basis of a topological space? What is a subbasis?
- (b) What is a subspace? What is the closure and the interior of a set? When is a set dense in a space?
- (c) What is a continuous function and when is it a homeomorphism? How can one prove that a map is continuous?
- (d) When is a space connected, path-connected, quasi-compact, compact, Hausdorff (T_2)? What are examples for spaces with or without these properties? What are components with respect to (path-)connectivity? What are important properties of compact spaces?
- (e) How is the product topology defined and what are projections? What do you know about properties of products?
- (f) What is initial and final topology? What are important examples of these? What are open sets in the initial/final topology? What is an embedding?
- (g) What is a metric space? When is a topological space first/second countable? When is a space metrisable?
- (h) What is a filter, ultrafilter and a filter basis? What are examples for filters? What does it mean for a filter to converge? What is a refinement and the image filter? What is an accumulation point of a filter? What are important properties of filters?
- (i) What is a T_i -space, $i \in \{1, 2, 3, 4\}$? What is a regular/normal/completely regular space? Which connections hold between these properties? What do you know about subspaces and products of T_i -spaces, $i \in \{1, 2, 3, 4\}$? What does Urysohn's Lemma and Tietze's Theorem state?

- (j) What is a compactification of a space? What is a morphism and which properties do morphisms have? When are two compactifications equivalent? Which special compactifications do you know? What are the prerequisites for the existence of a one-point compactification? What is the universal property of the Stone-Čech compactification?
- (k) What is a quotient map? How does one get a quotient map from an equivalence relation? How can you get a map $X/\sim \rightarrow Y$ from a map $X \rightarrow Y$?
- (l) How is identification of subspaces defined? Which important examples do you know? How do you attach a space to another? What are the canonical inclusion and the characteristic map of attached spaces?
- (m) What is a homotopy? What is homotopy relative to a subspace? What are important results about homotopic mappings? How can you get a homotopy on a quotient space X/\sim , given a homotopy on X ? What defines a contractible space? Which examples do you know?
- (n) What is w^- and $w * w'$ for paths w and w' ? What is a closed path or loop?
- (o) What are the elements of the fundamental group? How is the group operation defined and what is the neutral element? What is the induced homomorphism of a continuous map and what fundamental properties of it do you know?
- (p) What is the fundamental group of a disconnected space? What is the fundamental group of a contractible space? Which fundamental groups of spaces do you know?
- (q) What is a covering map? Which examples do you know? What important results are there about lifting paths in coverings?
- (r) What is the degree of a map $S^1 \rightarrow S^1$ and what are important properties?
- (s) What defines two homotopy-equivalent spaces? Can you give examples and useful statements? When is a space simply connected?
- (t) What does the Seifert-van Kampen Theorem state? How does the fundamental group of a space change if you attach a cell?
- (u) What is an abstract simplicial complex and what is its realisation? What are the characteristic maps? How do you compute the fundamental group of a space using a triangulation?
- (v) What are important homeomorphic invariants?



THE OFFICIAL END-OF-TOPOLOGY-CLASS “UMTRUNK”:
 AT CAFÉ HARDENBERG
 ON WED 11 FEB, STARTING AT 6PM

