Technische Universität Berlin
Fakultät II – Institut für Mathematik

Berlin Mathematical School
Wintersemester 2008/2009

Topology

Dr. Carsten Schultz

Axel Werner

Repetition

Question 86.

(a) What is a topological space? What defines open sets, closed sets and neighbourhoods? What is a (neighbourhood) basis of a topological space? What is a subbasis?

(b) What is a subspace? What is the closure and the interior of a set? When is a set dense in a space?

(c) What is a continuous function and when is it a homeomorphism? How can one prove that a map is continuous?

(d) When is a space connected, path-connected, quasi-compact, compact, Hausdorff (T_2)? What are examples for spaces with or without these properties? What are components with respect to (path-)connectivity? What are important properties of compact spaces?

(e) How is the product topology defined and what are projections? What do you know about properties of products?

(f) What is initial and final topology? What are important examples of these? What are open sets in the initial/final topology? What is an embedding?

(g) What is a metric space? When is a topological space first/second countable? When is a space metrisable?

(h) What is a filter, ultrafilter and a filter basis? What are examples for filters? What does it mean for a filter to converge? What is a refinement and the image filter? What is a accumulation point of a filter? What are important properties of filters?

(i) What is a T_i-space, $i \in \{1, 2, 3, 4\}$? What is a regular/normal/completely regular space? Which connections hold between these properties? What do you know about subspaces and products of T_i-spaces, $i \in \{1, 2, 3, 4\}$? What does Urysohn's Lemma and Tietze's Theorem state?

(j) What is a compactification of a space? What is a morphism and which properties do morphisms have? When are two compactifications equivalent? Which special compactifications do you know? What are the prerequisites for the existence of a one-point compacification? What is the universal property of the Stone-Čech compactification?

(k) What is a quotient map? How does one get a quotient map from an equivalence relation? How can you get a map $X/\sim \to Y$ from a map $X \to Y$?

(l) How is identification of subspaces defined? Which important examples do you know? How do you attach a space to another? What are the canonical inclusion and the characteristic map of attached spaces?

(m) What is a homotopy? What is homotopy relative to a subspace? What are important results about homotopic mappings? How can you get a homotopy on a quotient space $X/\sim$, given a homotopy on X? What defines a contractible space? Which examples do you know?

(n) What is w^- and $w * w'$ for paths w and w'? What is a closed path or loop?

(o) What are the elements of the fundamental group? How is the group operation defined and what is the neutral element? What is the induced homomorphism of a continuous map and what fundamental properties of it do you know?

(p) What is the fundamental group of a disconnected space? What is the fundamental group of a contractible space? Which fundamental groups of spaces do you know?

(q) What is a covering map? Which examples do you know? What important results are there about lifting paths in coverings?

(r) What is the degree of a map $S^1 \to S^1$ and what are important properties?

(s) What defines two homotopy-equivalent spaces? Can you give examples and useful statements? When is a space simply connected?

(t) What does the Seifert-van Kampen Theorem state? How does the fundamental group of a space change if you attach a cell?

(u) What is an abstract simplicial complex and what is its realisation? What are the characteristic maps? How do you compute the fundamental group of a space using a triangulation?

(v) What are important homeomorphic invariants?

The official end-of-topology-class "Umtrunk":
at Café Hardenberg
on Wed 11 Feb, starting at 6pm