TECHNISCHE UNIVERSITÄT BERLIN
Fakultät II – Institut für Mathematik

BERLIN MATHEMATICAL SCHOOL
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Topology

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Problem Set 2

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Exercise 8. **4 points**

Let X be a topological space. Show that the following statements are equivalent:

(a) X is a T_2-space.

(b) The diagonal $\Delta := \{(x,x) \in X \times X\}$ is closed in $X \times X$.

(c) For every space Z, every dense subset $D \subseteq Z$ and every two continuous maps $f, g : Z \to X$ such that $f|_D = g|_D$ we have $f = g$.

Exercise 9. **4 points**

Show:

(a) $\mathbb{R}$ is not homeomorphic to $\mathbb{R}^2$.

(b) S^1 is not homeomorphic to S^2.

Hint: Exercise 7.

Definition. Let X be a topological space, M a set and $f : X \to M$ a function. f is called *locally constant*, if every $x \in X$ has a neighbourhood U such that $f|_U$ is constant.

Exercise 10. **4 points**

Consider the following subspace of $\mathbb{R}^2$:

$$X := \left(\left\{\frac{1}{n} \;\middle|\; n \in \mathbb{N} \setminus \{0\}\right\} \times I\right) \cup \{(0,0), (0,1)\}$$

(a) Determine the connected components of X.

(b) Show that for every locally constant function $f : X \to M$ to some set M we have $f(0,0) = f(0,1)$.

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Exercise 11. **4 points**

Let $n \geq 2$ and A a countable subset of $\mathbb{R}^n$. Show that $\mathbb{R}^n \setminus A$ is path connected.

Exercise 12. **(Tutorial)**

Let X be a topological space, M a set and $f : X \to M$ a locally constant mapping. Show:

(a) Given any $y \in M$, the set $\{x \in X \mid f(x) = y\}$ is open and closed.

(b) If X is connected then f is constant.

Exercise 13. **(Tutorial)**

Let (X, d_X) and (Y, d_Y) be metric spaces and define

$$\begin{aligned} d_{X\times Y} : (X \times Y) \times (X \times Y) &\to \mathbb{R} \\ ((x, y), (x', y')) &\mapsto d_X(x, x') + d_Y(y, y'). \end{aligned}$$

Show:

(a) $(X \times Y, d_{X\times Y})$ is a metric space.

(b) The induced topology on $X \times Y$ by $d_{X\times Y}$ is the product topology of the topologies on X and Y, induced by d_X and d_Y, respectively.

Exercise 14. **(Tutorial)**

Let X and Y be topological spaces and $A \subset X$, $B \subset Y$ with the subspace topologies. Show that the product topology on $A \times B$ coincides with the subspace topology of $A \times B$ as a subset of $X \times Y$.

Exercise 15. **(Tutorial)**

Let X be a topological space and $A, B \subseteq X$ closed subsets such that $X = A \cup B$. Then a function $f : X \to Y$ is continuous if and only if both $f|_A$ and $f|_B$ are continuous.