Technische Universität Berlin
Fakultät II – Institut für Mathematik

Berlin Mathematical School
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Topology

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Problem Set 3

Deadline: 12 Nov 2008

Exercise 16. **4 points**

Let X and Y be quasi-compact spaces. Show that $X \times Y$ is again quasi-compact.
Hint: For a given open cover of $X \times Y$ and $x \in X$ show that there is a finite subcover of $\{x\} \times Y$. Then use Proposition 2.36 from the lecture and the quasi-compactness of X.

Exercise 17. **4 points**

Show that for any index set J and path connected spaces X_j, $j \in J$, the product space $\prod_{j \in J} X_j$ is again path connected.
Remark: If we replace "path connected" by "connected", then the statement is still true. For a finite index set J this has been proved in Proposition 2.14, for infinite J it is more complicated...

Exercise 18. **4 points**

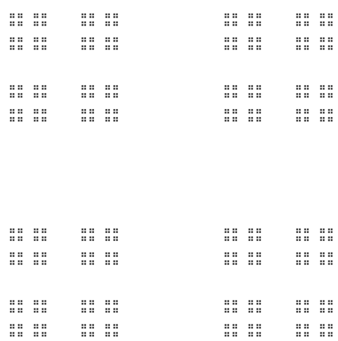
Let (X, d) be a metric space. Show that the topology on X induced by d is the initial topology with respect to the functions $f_y : X \to \mathbb{R}, x \mapsto d(x, y)$ for $y \in X$.

Exercise 19. **4 points**

Let $C \subset [0, 1]$ be the Cantor set.

(a) Show that $C \times C$ is homeomorphic to C.

(b) What are the connected components of C?



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Exercise 20. **(Tutorial)**

With $I = [0,1]$ show that I^I has a countable dense subset D.
Remark: This implies that the closure of D has more points than there are sequences in D!

Exercise 21. **(Tutorial)**

Let $S = (\{0,1\}, \{\emptyset, \{1\}, \{0,1\}\})$ be the Sierpiński space and $(X, \mathcal{T})$ some topological space.

(a) Show that $\mathcal{T}$ is the initial topology with respect to the functions

$$\chi_O : X \to S\,,\ \chi_O(x) = \begin{cases} 1 & \text{if } x \in O \\ 0 & \text{otherwise} \end{cases}\,, \qquad \text{for all } O \in \mathcal{T}$$

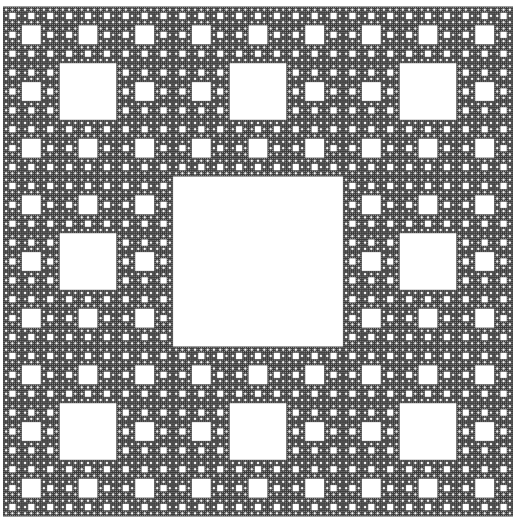
(b) What is the necessary condition for the function $\phi : X \to S^{\mathcal{T}}$, $\phi(x)(O) = \chi_O(x)$ to be an embedding?

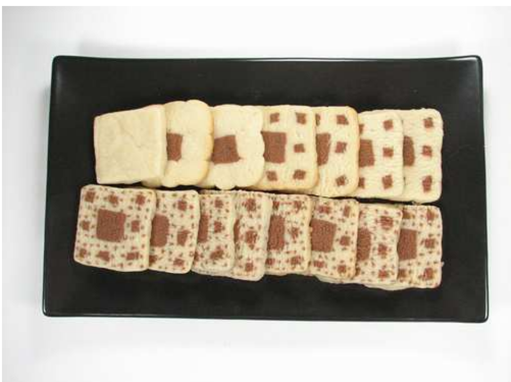
Exercise 22. **(Tutorial)**

Let M be a set and for $a \in M$ define $\delta_a \in \{0,1\}^M$ by

$$\delta_a(b) = \begin{cases} 1 & \text{if } a = b, \\ 0 & \text{otherwise.} \end{cases}$$

What is the closure of the set $\{\delta_a \mid a \in M\}$? Does it depend on M?





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