Solution to the Train Platforming Problem

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Plan

- Train Platforming Problem (TPP)
- Integer programming model
- Solution approach
- Computational results
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Train Platforming Problem

Input

- Train schedule: arrival, departure times, directions and allowed shifts
- Railway station topology: platforms, paths and directions

Output

- Assign each train a platform and two paths for arrival and departure s.t. no operational constraint is violated
## Train Schedule

<table>
<thead>
<tr>
<th>ID</th>
<th>Origin</th>
<th>Destination</th>
<th>Arrival</th>
<th>Departure</th>
<th>Shifts</th>
</tr>
</thead>
<tbody>
<tr>
<td>000001878</td>
<td>BOLOGNA C.LE</td>
<td></td>
<td>03.57.00</td>
<td>04.00.00</td>
<td>01 01 01 01 L</td>
</tr>
</tbody>
</table>

- **Arrival and Departure Times**
- **Directions and Preference List**
- **Allowed Shifts**
Railway Station Topology

Train Platforming Problem

- MILANO (1)
- PADOVA (2)
- FIRENZE (3)
- BOLOGNA C.LE.
- DEPOT (4)
Platform assignment

![Diagram showing platform assignment and platform conflicts]

- Train Platforming Problem
- Platform conflicts
- DEPOT (4)
- MILANO (1)
- PADOVA (2)
- FIRENZE (3)
- A, D, G
- B, E
- C, F
Paths assignment

A,D,G

path conflicts <= MAX_CONFL
Goals

- Assign each train a **platform**
  - **platform occupation**: at any instant each platform can be assigned to a single train

- Assign each train an **arrival and a departure path**
  - **path occupation**: incompatible paths can be assigned to different trains at overlapping time intervals if the overlapping windows are smaller than a given threshold
Preference list

- Each train has an arrival and a departure direction
- Trains with the same arrival and departure directions should be assigned to the same platform
- A pair of directions, respectively for arrival and departure, defines a preference list
- The preference list contains preference platforms for the specified pair sorted by preference score
Platforms assignment

- If a train cannot be assigned to a preference platform we use an out-list platform.
- If all platforms (paths) are busy we use a dummy platform.
There are two types of penalty:

1) Costs associated to **used platforms**

2) Penalties associated to **platforming “quality”**

\[
F_o = k_1 \cdot B_1 + k_2 \cdot B_2 + k_3 \cdot B_3 + k_4 \cdot B_4 + k_5 \cdot P_5 + k_6 \cdot P_6 + k_7 \cdot P_7 + k_8 \cdot P_8
\]

\* Trains are weighed by their priority
A pattern is a 5-uple:

\[ p(t) = (b, i_a, i_d, s_a, s_d) \]

where:

\( b \in B(t) \)

\( i_a \in IA(t) \quad i_d \in ID(t) \)

\( s_a \in SA(t) \quad s_d \in SD(t) \)
Variables

\[ y_b = \begin{cases} 
1 & \text{if platform } b \text{ is used} \\
0 & \text{otherwise} 
\end{cases} \quad b \in B \]

\[ x_{t,p} = \begin{cases} 
1 & \text{if train } t \text{ uses pattern } p \\
0 & \text{otherwise} 
\end{cases} \quad t \in T, p \in P(t) \]

\[ w_{t_1,t_2} = \text{number of minutes with path conflicts between the train pair } [t_1,t_2] \quad (t_1,t_2) \in NTT \]
Mathematical Model

\[
\begin{align*}
\text{min} & \sum_{b \in B} c_b y_b + \sum_{t \in T} \sum_{p \in P(t)} c_{t,p} x_{t,p} + \sum_{(t_1,t_2) \in NTT} c_{t_1,t_2} w_{t_1,t_2} \\
\sum_{p \in P(t)} x_{t,p} & = 1 \quad (1) \\
\sum_{(t,p) \in K(b,j)} x_{t,p} & \leq y_b \quad (2) \\
\sum_{(t,p) \in \gamma} x_{t,p} & \leq 1 \quad (3) \\
\sum_{p_1 \in P(t_1)} \xi_{p_1}(\chi) x_{t_1,p_1} & + \sum_{p_2 \in P(t_2)} \xi_{p_2}(\chi) x_{t_2,p_2} + \xi(\chi) & \leq w_{t_1,t_2} \quad (4) \\
y_b & \in \{0,1\}, \quad x_{t,p} \in \{0,1\}, \quad w_{t_1,t_2} \text{ integer} \quad (5)
\end{align*}
\]

- \( K(b,j) \) is the set of patterns that occupy platform \( b \) at instant \( j \)
- \( J \) is the set of all possible instants in which a platform occupation may start
Costs definition

The costs of the variables can be easily obtained from the objective function previously shown:

\[
\begin{align*}
\min \sum_{b \in B} c_b v_b + \sum_{t \in T} \sum_{p \in P(t)} c_{t,p} x_{t,p} + \sum_{(t_1,t_2) \in NT} c_{t_1,t_2} w_{t_1,t_2}
\end{align*}
\]

- \( c_b = K_1 + K_2 (b \notin PL(t)) + K_4 (b \in D) \)
- \( c_{t,p} = K_5 \cdot \text{totalshift} \cdot p_t + K_7 \cdot p_t (b \notin PL(t)) + K_8 \cdot p_t (b \in D) \)
- \( c_{t_1,t_2} = K_6 \cdot p_{t_1} \cdot p_{t_2} \)
Path-incompatibility constr.s

\[ \sum_{(t,p) \in \gamma} x_{t,p} \leq 1 \quad \gamma \in \Gamma_{(t_1,t_2) \in NTT} \]  

\( x_{t_1,p \in P(t_1)} \quad x_{t_2,p \in P(t_2)} \)

\[ G^3(t_1,t_2) \]

\[ 0.3 + 0.7 < 1 \]
\[ 0.5 + 0.7 > 1 \]
\[ 0.3 + 0.7 + 0.5 + 0.5 > 1 \]

Strengthening by clique-ing

Separation Problem for each \((t_1,t_2)\) pair:
- maximum Weight Clique Problem
- maximum Weight Stable Set
  on the complementary graph of \(G^3(t_1,t_2)\) which is bipartite

Max-Flow Min-Cut (polynomial)
Path-conflict constr.s:

\[ \sum_{p_{1} \in P(t_1)} \xi_{p_1}(\chi)x_{t_1,p_1} + \sum_{p_{2} \in P(t_2)} \xi_{p_2}(\chi)x_{t_2,p_2} + \xi(\chi) \leq w_{t_1,t_2} \]

\( \chi \in X_{(t_1,t_2) \in NTT} \) (4)

\[
\begin{align*}
5x_{t_1,p_1} + 5x_{t_2,p_2} - 5 & \leq w_{t_1,t_2} \\
2x_{t_1,p_3} + 2x_{t_2,p_2} - 2 & \leq w_{t_1,t_2} \\
3x_{t_1,p_1} + 3x_{t_2,p_4} - 3 & \leq w_{t_1,t_2} \\
6x_{t_1,p_3} + 6x_{t_2,p_4} - 6 & \leq w_{t_1,t_2}
\end{align*}
\]
Path-conflict constr.s

\[
\sum_{p \in P(t_1)} \xi_{p_1}(\chi)x_{t_1,p_1} + \sum_{p \in P(t_2)} \xi_{p_2}(\chi)x_{t_2,p_2} + \xi(\chi) \leq w_{t_1,t_2}
\]

Polytope description:

\[
\{(e_i, e_j, c_{i,j}) : i \in \{1,.., m\}, j \in \{1,.., n\}\}
\]

The number of vertices is mn, that is polynomial with respect to the problem dimension, thus separation can be performed polynomially as well.

The separation problem is the following: given a point \((x^{*t_1}, x^{*t_2}, w^*)\) does it belong to P?

Taking the dual and applying Farkas Lemma we obtain the following separation problem:

\[
\max \sum_{i=1}^{m} \alpha_i x_i^{t_1} + \sum_{j=1}^{n} \beta_j x_j^{t_2} + \delta w_{t_1,t_2} + \gamma
\]

s.t.

\[
\alpha_i + \beta_j + \gamma + c_{i,j} \delta \leq 0 \quad \forall \ i = 1,.., m \quad j = 1,.., n
\]
Solution methodology

- The algorithm implemented is a Branch-and-Price procedure based on the continuous relaxation of the ILP model.
Solution methodology

- ...but the model has a huge number of variables and constraints
- It is impossible to handle them directly using a general purpose solver for LP models, so...

Column generation

Separation
A “cut-and-price” approach

- rows…
  - constraints (1)
  - constraints (2)
  - constraints (3)
  - constraints (4)

- …and columns

\[
\begin{align*}
\mathbf{y_b} & \quad \mathbf{x_{t,p}} & \quad \mathbf{w_{t,j_2}}
\end{align*}
\]
Computational results

- CPLEX 10
- Pentium 4 3.2 GHz
- OS: Windows XP Pro
- 1 GB DDR SDRAM
- C language
## Computational results

### PALERMO C.LE

<table>
<thead>
<tr>
<th>station</th>
<th>PA_C.LE</th>
<th>K1</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td># trains</td>
<td>204</td>
<td>K2</td>
<td>50</td>
</tr>
<tr>
<td># platforms</td>
<td>11</td>
<td>K3</td>
<td>0</td>
</tr>
<tr>
<td># directions</td>
<td>4</td>
<td>K4</td>
<td>99000</td>
</tr>
<tr>
<td># paths</td>
<td>64</td>
<td>K5</td>
<td>1</td>
</tr>
<tr>
<td># pairs of incompatible paths</td>
<td>1182</td>
<td>K6</td>
<td>5</td>
</tr>
<tr>
<td># maximum travel time on a path</td>
<td>3</td>
<td>K7</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>K8</td>
<td>10000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>instance ID</th>
<th>MAXCNFL</th>
<th>S_HEUR</th>
<th>S_LP</th>
<th>B_S_ILP</th>
<th>time</th>
</tr>
</thead>
<tbody>
<tr>
<td>PA C.LE.</td>
<td>0</td>
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<td>449044</td>
<td>200</td>
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<td>230</td>
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<td>PA C.LE.</td>
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<td>380182</td>
<td>10159</td>
<td>10172</td>
<td>339</td>
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### Computational results

#### BARI C.LE

<table>
<thead>
<tr>
<th>station</th>
<th>BA_C.LE</th>
<th>K1</th>
<th>1000</th>
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</thead>
<tbody>
<tr>
<td># trains</td>
<td>237</td>
<td>K2</td>
<td>50</td>
</tr>
<tr>
<td># platforms</td>
<td>14</td>
<td>K3</td>
<td>0</td>
</tr>
<tr>
<td># directions</td>
<td>5</td>
<td>K4</td>
<td>99000</td>
</tr>
<tr>
<td># paths</td>
<td>89</td>
<td>K5</td>
<td>1</td>
</tr>
<tr>
<td># pairs of incompatible paths</td>
<td>1996</td>
<td>K6</td>
<td>5</td>
</tr>
<tr>
<td># maximum travel time on a path</td>
<td>4</td>
<td>K7</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>K8</td>
<td>10000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>instance ID</th>
<th>MAXCNFL</th>
<th>S_HEUR</th>
<th>S_LP</th>
<th>B_S_ILP</th>
<th>time</th>
</tr>
</thead>
<tbody>
<tr>
<td>BA C.LE.</td>
<td>0</td>
<td>1576300</td>
<td>653264</td>
<td>808255</td>
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<td>BA C.LE.</td>
<td>1</td>
<td>1398330</td>
<td>373486</td>
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<tr>
<td>BA C.LE.</td>
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<td>1197485</td>
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<td>BA C.LE.</td>
<td>3</td>
<td>838235</td>
<td>8885</td>
<td>8924</td>
<td>270</td>
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</table>
## Computational results

### GENOVA P.PRINC.

<table>
<thead>
<tr>
<th>station</th>
<th>GENOVA P.PRINC.</th>
<th>K1</th>
<th>K2</th>
<th>K3</th>
<th>K4</th>
<th>K5</th>
<th>K6</th>
<th>K7</th>
<th>K8</th>
</tr>
</thead>
<tbody>
<tr>
<td># trains</td>
<td>127</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># platforms</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># directions</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># paths</td>
<td>174</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td># pairs of incompatible paths</td>
<td>7154</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># maximum travel time on a path</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>instance ID</th>
<th>MAXCNFL</th>
<th>S_HEUR</th>
<th>S_LP</th>
<th>B_S_ILP</th>
<th>time</th>
</tr>
</thead>
<tbody>
<tr>
<td>SE P.PRINC.</td>
<td>0</td>
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