Modeling and solving a multimodal multicapacitated routing problem with scheduled services, time windows, and economies of scale

Luigi Moccia\textsuperscript{1,2}, Jean-François Cordeau\textsuperscript{3}, Gilbert Laporte\textsuperscript{4}, Stefan Ropke\textsuperscript{4}, and Maria Pia Valentini\textsuperscript{1}

\textsuperscript{1}ENEA - Ente per le Nuove tecnologie, l’Energia e l’Ambiente, Italy
\textsuperscript{2}Università della Calabria, DEIS, Italy
\textsuperscript{3}Canada Research Chair in Logistics and Transportation, Canada
\textsuperscript{4}Canada Research Chair in Distribution Management, Canada

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Outline

1. Introduction
   - Problem characteristics
   - Relevant application related literature

2. Modelling Approach
   - Node-Arc based Formulation - $F_1$
   - Path based Formulation - $F_2$
   - Approximated Path based Formulation - $F_2L$
   - Valid inequalities

3. Algorithms and Computational results
   - Algorithms
   - Column generation details
   - Dynamic slope scaling, algorithm CGDSS
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Moccia et al.  Multimodal Routing
Operational problem faced by freight forwarders in multimodal service network
- Consolidation of shipments: economies of scale
- Mix of flexible-time and scheduled transportation services
- Multiple time windows
- Multiple capacity constraints, e.g. volume, weight, train length, etc.
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Moccia et al.  Multimodal Routing
Modelling approach


- PLUS: TWs and PL.
Figure: Portion of the digraph representing the pickup and delivery time windows for a commodity
**Figure:** Example of exploding the physical network to the virtual one
We model the M++RP as a ODIMCFP with Time Windows (some of them collapsed).

This formulation employs binary variables $x_{ij}^k, \forall (i,j) \in A, \forall k \in K$, where $x_{ij}^k = 1$ if commodity $k$ uses arc $(i,j)$, $x_{ij}^k = 0$ otherwise.

$T_{ik}^k \in \mathbb{R}^+, \forall k \in K, \forall i \in I$, arrival time of commodity $k$ at node $i$; $T_{ik}^k > 0$ if the node is visited by the commodity, otherwise $T_{ik}^k = 0$. 
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\( T^k_i \in \mathbb{R}^+, \forall k \in K, \forall i \in I \), arrival time of commodity \( k \) at node \( i \); \( T^k_i > 0 \) if the node is visited by the commodity, otherwise \( T^k_i = 0 \).
\[
\sum_{j \in \Omega(k) \cup \{d^k\}} x^k_{o^k, j} = 1 \quad \forall k \in K, \quad (1)
\]
\[
\sum_{i \in \Gamma(k) \cup \{o^k\}} x^k_{i, d^k} = 1 \quad \forall k \in K, \quad (2)
\]
\[
\sum_{j \in \delta^+(i)} x^k_{i, j} - \sum_{j \in \delta^-(i)} x^k_{j, i} = 0 \quad \forall k \in K, \forall i \in I, \quad (3)
\]
\[
T^k_i + t^k_{ij} - T^k_j \leq (1 - x^k_{ij}) M^k_{ij} \quad \forall k \in K, \forall (i,j) \in A \cap I \times I, (4)
\]
\[
a^i \sum_{j \in \delta^+(i)} x^k_{i, j} \leq T^k_i \leq b^i \sum_{j \in \delta^+(i)} x^k_{i, j} \quad \forall k \in K, \forall i \in N^{tw}, \quad (5)
\]
\[
T^k_i = d^i \sum_{j \in \delta^+(i)} x^k_{i, j} \quad \forall k \in K, \forall i \in N^d, \quad (6)
\]
\[
\sum_{k \in K} w^k_{ij} x^k_{ij} \leq W_{ij} \quad \forall (i,j) \in A^{pl}. \quad (7)
\]
We use the so called *Multiple Choice Model* (MCM) to represent these PL functions. The MCM uses the following variables:

- \( l_{ij}^s \in \mathbb{R}^+ \), \( \forall (i, j) \in A_{pl} \), \( \forall s \in S_{ij} \), expresses the arc load on segment \( s \); if \( l_{ij}^s > 0 \) implies \( l_{ij}^u = 0 \), \( \forall u \in S_{ij} \setminus \{s\} \) and \( r_{ij}^{s-1} \leq l_{ij}^s \leq r_{ij}^s \);

- \( y_{ij}^s \in \{0, 1\} \), \( \forall (i, j) \in A_{pl} \), \( \forall s \in S_{ij} \), where \( y_{ij}^s = 1 \) if the arc load belongs to the segment \( s \) of the cost function \( g_{ij} \); otherwise \( y_{ij}^s = 0 \).
\[
\begin{align*}
\min & \sum_{k \in K} \sum_{(i,j) \in A^v} c_{ij}^k x_{ij}^k + \sum_{(i,j) \in A^{pl}} \sum_{s \in S_{ij}} (c_{ij}^s l_{ij}^s + f_{ij}^s y_{ij}^s) \\
\text{subject to } (1)-(7), \text{ and } & \\
\sum_{s \in S_{ij}} l_{ij}^s = \sum_{k \in K} q_{ij}^k x_{ij}^k & \forall (i,j) \in A^{pl}, \\
\frac{r_{ij}^{s-1}}{y_{ij}} \leq l_{ij}^s \leq r_{ij}^s y_{ij} & \forall (i,j) \in A^{pl}, \forall s \in S_{ij}, \\
\sum_{s \in S_{ij}} y_{ij}^s \leq 1 & \forall (i,j) \in A^{pl}.
\end{align*}
\]
Path based Formulation - $\mathcal{F}_2$

- We assume that for each commodity $k$ is defined a path $p$ between the origin and the destination nodes over the digraph $G$.
- Finding a minimum cost path $p$ is a special case of resource constrained path problems.
- In this formulation we have binary decision variables $z^k_p$, where $z^k_p = 1$ if the commodity $k$ is routed by the path $p$, and equals zero otherwise.
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Finding a minimum cost path $p$ is a special case of resource constrained path problems.

In this formulation we have binary decision variables $z^k_p$, where $z^k_p = 1$ if the commodity $k$ is routed by the path $p$, and equals zero otherwise.
min \sum_{k \in K} \sum_{p \in P(k)} c^k_p z^k_p + \sum_{(i,j) \in A^p} \sum_{s \in S_{ij}} (c^s_{ij} l^s_{ij} + f^s_{ij} y^s_{ij}) \quad (12)

subject to (10), (11), and

\sum_{p \in P(k)} z^k_p = 1 \quad \forall k \in K, \quad (13)

\sum_{s \in S_{ij}} l^s_{ij} - \sum_{k \in K} \sum_{p \in P(k)} q^k_{ij} \phi^p_{ij} z^k_p = 0 \quad \forall (i, j) \in A^p, \quad (14)

\sum_{k \in K} \sum_{p \in P(k)} w^k_{ij} \phi^p_{ij} z^k_p \leq W_{ij} \quad \forall (i, j) \in A^p, \quad (15)

z^k_p \in \{0, 1\} \quad \forall k \in K, p \in P(k). \quad (16)
Approximated Path based Formulation - $F_2L$

A piecewise linear cost function will be approximated by its lower convex envelope when relaxing integrality constraints in a MILP formulation:

\[
\begin{align*}
\min & \sum_{k \in K} \sum_{p \in P(k)} \left( \sum_{(i,j) \in A^v} c_{ij}^k \psi_{ij}^p \right) + \sum_{(i,j) \in A^p} \alpha_{ij} q_{ij}^k \phi_{ij}^p z_k^p \\
\text{subject to} \ (13), \ (15), \ \text{and:} \\
& \sum_{k \in K} \sum_{p \in P(k)} q_{ij}^k \phi_{ij}^p z_k^p \leq r_{ij} \left| S_{ij} \right| \quad \forall (i, j) \in A^p
\end{align*}
\]
Valid inequalities

We adapt to formulations $\mathcal{F}_1$ and $\mathcal{F}_2$ the valid inequalities proposed by:


The *strong forcing constraints* state that when on a given arc no segment is chosen, the flow of each commodity is zero on that arc.

\[
x_{ij}^k \leq \sum_{s \in S_{ij}} y_{ij}^s \quad \forall k \in K, \forall (i, j) \in A^{pl} \quad (19)
\]
The extended forcing constraints require additional non-negative variables \( x_{ij}^{ks} \), and:

\[
x_{ij}^k = \sum_{s \in S_{ij}} x_{ij}^{ks} \quad \forall k \in K, \forall (i, j) \in A^{pl},
\]

(20)

\[
l_{ij}^s = \sum_{k \in K} q_{ij}^k x_{ij}^{ks} \quad \forall (i, j) \in A^{pl}, \forall s \in S_{ij},
\]

(21)

\[
x_{ij}^{ks} \leq y_{ij}^s \quad \forall (i, j) \in A^{pl}, \forall k \in K, \forall s \in S_{ij}.
\]

(22)

Then, we have formulations \( \mathcal{F}_1 S, \mathcal{F}_1 E, \mathcal{F}_2 S \) and \( \mathcal{F}_2 E \).
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Algorithms:

- the cplex implementation of formulations $\mathcal{F}_1$, $\mathcal{F}_1S$, $\mathcal{F}_1E$;
- CG1: the column generation that solves $\mathcal{F}_2L$ and looks for a feasible integer solution with the cplex MILP solver out of the pool of generated columns, i.e. the Restricted Master Problem;
- CG2: the column generation for formulation $\mathcal{F}_2$ and the cplex MILP solver on the RMP;
- CGS: column and row generation for formulation $\mathcal{F}_2S$, plus the cplex MILP solver on the RMP.
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The pricing phase must solve for each commodity a Shortest Path Problem with Time Windows and Timetables (SPPTWT).

Thanks to our representation of timetables as collapsed time windows, the SPPTWT is treatable with any SPPTTW algorithm.

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Test instances

Realistic instance
- 122 shipments from 10 factories to 10 regional distribution centers (from North to South Italy), two weeks time horizon, 28 block trains available;

Smaller test instances
- subsets of real size instance with $\bar{k}$ equal to 10, 30, 60.
Test instances

Realistic instance

- 122 shipments from 10 factories to 10 regional distribution centers (from North to South Italy), two weeks time horizon, 28 block trains available;
- resulting graph: $|N| = 1507$, $|A| = 4900$, $|A^{pl}| = 2094$.

Smaller test instances

subsets of real size instance with $\bar{k}$ equal to 10, 30, 60.
### CG1 Upper bound quality - part I

<table>
<thead>
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<th>Value</th>
<th>Time (min.)</th>
<th>Sol.</th>
<th>Value</th>
<th>Time (min.)</th>
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<td><strong>Average</strong></td>
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<td><strong>Average</strong></td>
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</table>
CG1 Upper bound quality - part II

“best LB” (days of computation!) by $F_1$, $F_1S$, $F_1E$, and CGS

<table>
<thead>
<tr>
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<tr>
<td>average</td>
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</table>
Other comp. results

- $\mathcal{F}_1 E$ hits memory availability too early ($\bar{k} = 60$);
- $\mathcal{F}_1$ vs. $\mathcal{F}_1 S$: lighter the better (in average);
- CGS useful to obtain better LBs, not competitive with CG1 for UBs;
- CGS more effective than $\mathcal{F}_1 S$ on medium-large instances.
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### $F_2L$ merit

<table>
<thead>
<tr>
<th>Instance</th>
<th>$F_1$ (sec.)</th>
<th>CG1 (sec.)</th>
<th>CG2 (sec.)</th>
<th>CG1/$F_1$</th>
<th>CG2/$F_1$</th>
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### CG1 Lower Bound quality

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<td>108</td>
</tr>
<tr>
<td>average</td>
<td>107</td>
<td>102</td>
</tr>
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</table>
CGDSS solves $\mathcal{F}_2L$ as CG1, then creates perturbed new problems by a dynamic slope scaling concept to enlarge the column pool. Finally, it starts the cplex MILP solver.


Figure: Example of slope updating between iteration zero and one
CGDSS is able to obtain a new better upper bound on the large i-122-01 instance (-0.2%).
Future work

- Branch-and-price
- New problem variants: time related node costs...
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