Prof. John Sullivan
Isabella Thiesen

Homework Assignment 6

Homework due May 29th after the tutorial.

Topics: Ruled surface, Shape operator, curvature, length-preserving parametrization

Problem 1

a) Compute the Shape operator for

   (a) the *catenoid*: $f(u,v) = (\cosh u \cos v, \cosh u \sin v, u)$ and

   (b) the *helicoid*: $f(u,v) = (\sinh u \cos v, \sinh u \sin v, v)$.

b) Show that the two surfaces have identical Gaussian curvature.

c) Show that the mean curvature of each vanishes. (Surfaces with mean curvature $H = 0$ are called *minimal surfaces*.)

(2+1+1points)

Problem 2

Let

$$f : \mathbb{R}^2 \to \mathbb{R}, \quad f(x,y) := (x, y, xy)$$

be a parametrization of the hyperbolic paraboloid.
Show the following: There exist two straight lines through each point $f(x,y)$ that lie on the surface. Conclude that the hyperbolic paraboloid is a *ruled surface*.
(4 points)

Problem 3

A parameterization $f : U \to \mathbb{R}^3$ is called *length-preserving* if it preserves the length of curves, i.e. for every curve $\gamma : [a,b] \to U$ with $c := f \circ \gamma$ holds $L(\gamma) = L(c)$ for the lengths of these curves.

a) Give a length-preserving parametrization of the cylinder.

b) Show that a parametrization is length-preserving if and only if $g_s ij = \delta_{ij}$ (the first fundamental form being the identity matrix).

(2+2 points)

Problem 4

Let $F : U \to \mathbb{R}^3$ be a regular surface element with principal curvatures κ_1 and κ_2 and normal vector ν. Let $F_c = F + c\nu$ be the *parallel surface* of F in distance c with $|c| < \min(|1/\kappa_1|, |1/\kappa_2|)$. Show that the Gaussian and mean curvature of F_c are given by

$$K_c = \frac{K}{1 - 2cH + c^2K}$$

$$H_c = \frac{H - cK}{1 - 2cH + c^2K}.$$

(2+2 points)