TU Berlin
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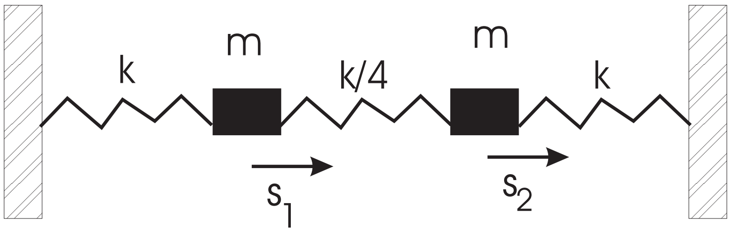
Mathematical Physics I
Exercises 1

Problem 1: Example for a dynamical system
Consider the vibrations of a two masses m and three elastic springs (moduli k, $k/4$ and k). Two of the springs are fixed to walls (see figure). Friction can be neglected, and the masses move in one dimension only.



The coordinates s_1 and s_2, respectively, denote the deviation of the masses from the position, where the springs are stress-free. The aim here is an equation of motion for theses coordinates and a solution. The initial conditions are:

$$\begin{aligned} s_1(0) = s_1^0 \ , \quad s_2(0) = s_2^0 \\ \dot{s_1}(0) = v_1^0 \ , \quad \dot{s_2}(0) = v_2^0 \ . \end{aligned} \tag{1}$$

(a) On each mass there are acting two forces. Formulate these forces in terms of the coordinates s_1 and s_2.

(b) Formulate Newton's law for each mass. This results in a system of coupled differential equations for s_1 and s_2.

(c) In order to decouple the differential equations, we introduce $\tilde{s}_1 := s_1 + s_2$ and $\tilde{s}_2 := s_1 - s_2$ and derive differential equations for these new coordinates.

(d) Solve the initial value problem for $\tilde{s}_1$ and $\tilde{s}_2$.

(e) Write the solution in terms of the original coordinates s_1 and s_2.

(f) Formulate the problem as a dynamical system on the phase space.

(10 points)

A differential equation does not lead automatically to a dynamical system, as we show in the following two examples:

Problem 2:
Prove that the following initial value problem has no unique solution:

$$\begin{aligned} \dot{x}(t) = x(t)^{1-\varepsilon} \ , \qquad \varepsilon \in (0,1) \ , \qquad t \in \mathbb{R} \\ x(0) = 0 \end{aligned} \tag{2}$$

Hint: You can find one solution by separation of the variables.

(5 points)

Problem 3:
Prove that the solution of the following initial value problem

$$\begin{aligned} \dot{x}(t) = 1 + x(t)^2 \ , \qquad t \in \mathbb{R} \\ x(0) = 1 \end{aligned} \tag{3}$$

exists only on a finite time interval. (5 points)

Deadline: Tuesday, October 30