# Regular and Semi-regular Polyhedra

## Visualization and Implementation

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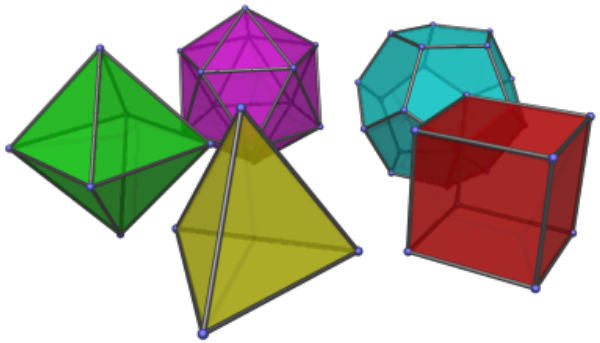
# Properties

- A polyhedron is called **regular** if all its faces are equal and regular polygons.
  It is called **semi-regular** if all its faces are regular polygons and all its vertices are equal.
- A regular polyhedron is called **Platonic solid**, a semi-regular polyhedron is called **Archimedean solid**.*



*in the following presentation we will leave out two polyhedra of the Archimedean solids

# Properties

- Dualizing a polyhedron interchanges its vertices and faces , whereas the dual edges are orthogonal to the edges of the polyhedron.
  The dual polyhedra of the Archimedean solids are called **Catalan solids**.
- in the following, we will take a closer look at those three polyhedra: Platonic, Archimedean and Catalan solids

# Properties

For a Platonic,Archimedean and Catalan solids the following statement holds:

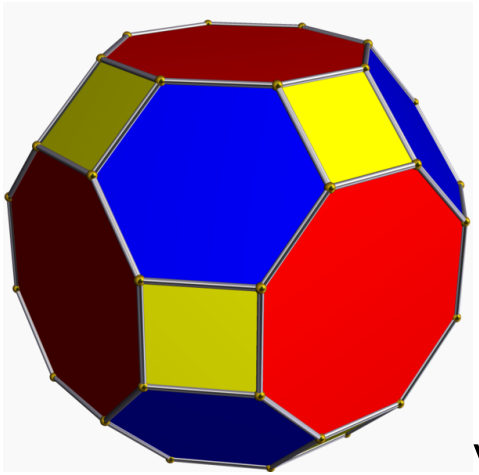
- all its vertices lie on one sphere

For an Archimedean or Platonic solid we know that :

- all its faces are regular polygons, the edges of the polygons all have same length
- all vertices are surrounded by the same number of faces, even by the same n-gons
- truncating the vertices or vertices and edges of a Platonic solid in a regular way gives an Archimedean solid

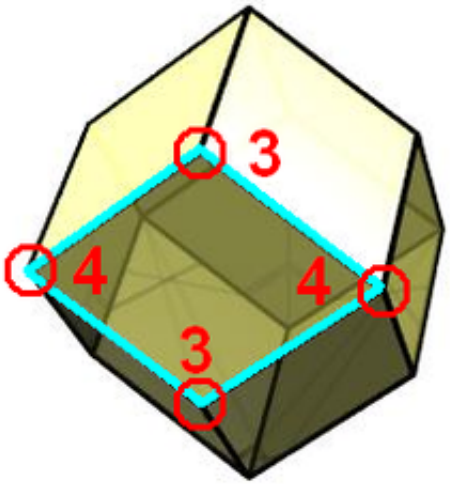
# Notations for Polyhedra

- vertex configuration : applicable for polyhedra with equal vertices, i.e. for Platonic and Archimedean solids
  for a vertex v it determines the number of sides of each polygon at v


vertex configuration 4.6.8

# Notations for Polyhedra

- face configuration : applicable for polyhedra with equal faces, i.e. Catalan solids
  for a face S it determines the number of faces lying around each vertex of S




V3.4.3.4

# Notations for Polyhedra

- notation in the project: use a Platonic solid as a seed, apply operations on it.
  the following operations are possible: truncating edges , truncating vertices, dualizing

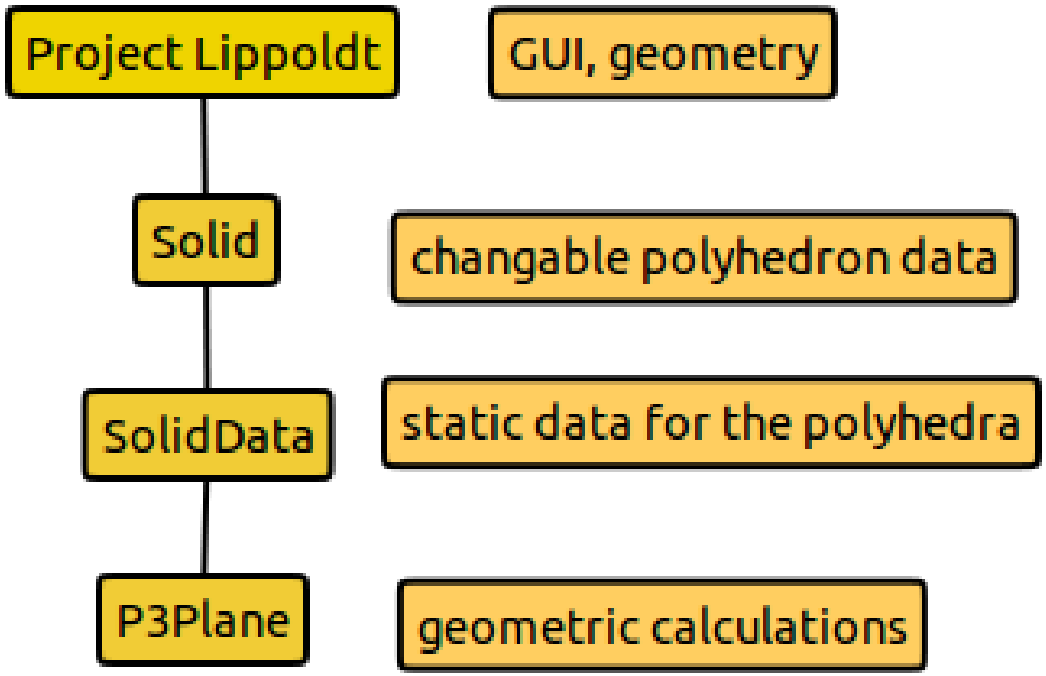
# Task

- visualize polyhedra by using a plane model
- find out how to implement and change those planes
- create an interface which explains the user how the planes work

# Plane Model

- create the polyhedra by setting up a fundamental region and using one of the triangle groups *233, *234, *235
- this fundamental region is surrounded by 3 planes, further planes cut the inner surface
- planes for cutting the surface are tangent planes of the sphere moved along the line from the origin to the tangent point
- 3 planes are enough to create all Platonic and Archimedean solids
- for the Catalan solids different planes are needed

# Structure and Classes



# Evaluation

unsolved problems:

1. to adjust the size of the polyhedra such that they have the same size
2. to dualize not only Archimedean solids but also other truncated Platonic solids
3. to make calculations easier

# Polyhedra in Real Space


- H.S.M. Coxeter, "Regular Polytopes" , 1963 Dover Publications
- P.R. Cromwell, "Polyhedra", 1997 Cambridge University Press
- Images taken from : http://eusebeia.dyndns.org/4d/platonic and http://en.wikipedia.org

# Any questions?