Exercise 1. The 13 Configuration

The above figure in $\mathbb{R}P^1$ consists of 13 points and 13 lines. Label the remaining points (marked with a letter or a small square) in this figure with their coordinates. Label the lines with their equations.

Exercise 2. Involutions in $\mathbb{R}P^1$

Given a projective transformation $f$ of $\mathbb{R}P^1$. Show that $f$ is the composition of three projective involutions.


Let $\ell, \ell'$ be two lines in a projective plane and $p := \ell \cap \ell'$. Let $a, b, c$ and $a', b', c'$ be distinct points on $\ell$, resp. on $\ell'$, all of them different from $p$. Prove that

$$\overline{aa'} \cap \overline{bb'} \cap \overline{cc'} \neq \emptyset \iff \operatorname{cr}(a, b, c, p) = \operatorname{cr}(a', b', c', p).$$

Exercise 4. Geometric construction of the sum of two points.

Let $x, y$ be two distinct points on $\mathbb{R} \cup \{\infty\} = \mathbb{R}P^1$, different from 0 and $\infty$. Construct the point $x + y$ geometrically, using complete quadrilaterals.

Due before the first lecture on January 12th, 2014