Exercise Sheet 8

Exercise 1. Incidence of Points and Planes

The ambient space for this problem is $\mathbb{R}P^3$.

a) Show that three non-collinear points determine a unique plane containing the three points.

b) Show that three planes not all containing the same line, have a unique point of intersection.

c) Find the plane equation for the plane containing the three points $(1, 1, 0, 1), (0, -1, 1, 1),$ and $(1, 0, 1, 1)$.

d) Show that two lines intersect in a point if and only if they lie in a plane.

Exercise 2. Projective transformations

Can a projective transformation $\mathbb{R}P^1 \rightarrow \mathbb{R}P^1$ which is not the identity have three fixed points? How many fixed points can a projective transformation $\mathbb{R}P^1 \rightarrow \mathbb{R}P^1$ have? Give an example for each possible case.

Exercise 3. Desargues’ theorem in 3D

Let $P_1, P_2, P_3, P_4$ and $Q_1, Q_2, Q_3, Q_4$ be two quadruples of pairwise distinct points in $\mathbb{R}P^3$. Then $P_1, ..., P_4$ and $Q_1, ..., Q_4$ form two tetrahedra. Suppose that the four lines $P_iQ_i$ intersect in one point. Show that the lines in which planes of corresponding faces intersect (e.g. $P_1P_2, P_3P_4$ and $Q_1Q_2, Q_3$) lie in a common plane.

Exercise 5. Pappus’ theorem

Let $P_1, P_2, P_3, P_4, P_5, P_6$ be distinct points in the projective plane $\mathbb{R}P^2$. Suppose that the three lines $P_1P_2, P_4P_5, P_3P_6$ intersect in one point, and the three lines $P_2P_3, P_5P_6, P_4P_1$ intersect in one point. Show that the lines $P_3P_4, P_6P_1, P_5P_2$ also intersect in one point. See figure on next page. (Tip: Find Pappus.)