5. Exercise Sheet – Topology

To be handed in on November 24 after the first lecture.

**Homework exercise 1** 5 points
Construct a continuous map $f: S^1 \times S^1 \to \mathbb{R}^2 \setminus \{0\}$ with $f(-x, y) = -f(x, y) = f(x, -y)$.

**Homework exercise 2** 5 points
Prove that removing any point from $\mathbb{R}P^2$ yields a space homotopy equivalent to $S^1$.

**Homework exercise 3** 5 points
Find an embedded torus $T$ in $\mathbb{R}P^3$ such that $\mathbb{R}P^3 \setminus T$ consists of two connected components both of which are homeomorphic to $S^1 \times D^2$. Here $D^2$ denotes the open 2-disk.

**Homework exercise 4** 5 points
Let $\Delta = \{(x, y) \in \mathbb{R}^2 \mid x \geq 0, y \geq 0, x + y \leq 1\}$ and let $\sim$ be the equivalence relation generated by $(x, 0) \sim (0, x)$ and $(x, 0) \sim (x, 1 - x)$. The topological space $D = \Delta/\sim$ is called *dunce hat*. Let $x_0 = [(0, 0)] \in D$. Find a 1-dimensional CW-complex homotopy equivalent to $D \setminus \{x_0\}$. 