6. Exercise Sheet – Topology

To be handed in on December 1 after the first lecture.

**Homework exercise 1** 5 points
Let $X$ be a path-connected space with path-connected open subsets $A, B \subseteq X$. Suppose $X = A \cup B$, $A$ and $B$ are not simply connected, but $A \cap B$ is simply connected. Show that for any $x_0 \in A \cap B$ and any loop $\gamma$ at the basepoint $x_0$ that is not homotopic to a constant loop, there is a loop $\gamma'$ based at $x_0$ such that $\gamma \cdot \gamma' \not\simeq \gamma' \cdot \gamma$.

**Homework exercise 2** 5 points
Let $X$ be a path-connected Hausdorff space and $p \in X$ a point with an open neighborhood $U \cong \mathbb{R}^d$ for some $d \geq 3$. Then for any $x_0 \in X \setminus \{p\}$ we have $\pi_1(X, x_0) \cong \pi_1(X \setminus \{p\}, x_0)$.

**Homework exercise 3** 5 points
Compute the fundamental group of $\mathbb{R}P^d$ for every integer $d \geq 1$.

**Homework exercise 4** 5 points
Compute the fundamental group of $S^2 \cup \{(x, y, z) \in \mathbb{R}^3 \mid x = y = 0\}$. 