10. Exercise Sheet – Topology

To be handed in on January 19 after the first lecture.

**Homework exercise 1** \hspace{0.5cm} 5 points
For every $n$ construct a $\Delta$-complex homeomorphic to $\mathbb{R}P^n$.

**Homework exercise 2** \hspace{0.5cm} 5 points
Compute the simplicial homology groups of some $\Delta$-complex homeomorphic to $\mathbb{R}P^2$ and $S^1 \vee S^1$.

**Homework exercise 3** \hspace{0.5cm} 5 points
The $n$-simplex $\Delta^n$ together with its faces forms a $\Delta$-complex.

Let $X$ be a subcomplex of the $n$-simplex $\Delta_n$. Then $H_{n-1}(X)$ is non-trivial if and only if $X = \partial\Delta^n$.

**Homework exercise 4** \hspace{0.5cm} 5 points
(a) Construct a $\Delta$-complex $X$ with $H_1(X) \cong \mathbb{Z}/4$ and $H_i(X) = 0$ for $i \geq 2$.
(b) Construct a $\Delta$-complex $X$ with $H_1(X) = 0$, $H_2(X) = \mathbb{Z}^2$, and $H_i(X) = 0$ for $i \geq 3$. 