12. Exercise Sheet – Topology

To be handed in on February 2 after the first lecture.

Homework exercise 1 5 points
Let $X$ be an annulus and $A \subseteq X$ its two boundary curves. Let $Y$ be a Möbius strip and $B \subseteq Y$ its boundary curve. Compute the relative homology groups of the pairs $(X, A)$ and $(Y, B)$ and compare them to the homology groups of the quotients $X/A$ and $Y/B$.

Homework exercise 2 5 points
Let $G$ be an abelian group that fits into the short exact sequence $\mathbb{Z}/4 \to G \to \mathbb{Z}/4$. Determine all possible isomorphism types of $G$. Which of these abelian groups also fit into the short exact sequence $\mathbb{Z}/2 \to G \to \mathbb{Z}/8$?

Hint: Recall the fundamental theorem of finitely generated abelian groups.

Homework exercise 3 5 points
Let $A$ be a finite point set on the torus $S^1 \times S^1$. Compute $H_n(S^1 \times S^1, A)$.

Homework exercise 4 5 points
Find an example of spaces $X$ and $Y$ and $A \subseteq X$ and $B \subseteq Y$ with $H_i(X) \cong H_i(Y)$ and $H_i(A) \cong H_i(B)$ for all $i$, but there is an $i$ such that $H_i(X, A) \not\cong H_i(Y, B)$. 