13. Exercise Sheet – Topology

To be handed in on February 9 after the first lecture.

Homework exercise 1 5 points
Let \( A = \{ \frac{1}{n} \mid n \in \mathbb{Z}, n > 0 \} \cup \{0\} \). Show that \( H_1([0,1]/A) \) and \( H_1([0,1], A) \) are not isomorphic.

Hint: Hawaiian earrings

Homework exercise 2 5 points
Let \( X \) be a non-empty space. Compute the homology groups of the cone \( CX \).

Homework exercise 3 5 points
Let \( X \) be a space. Show that \( H_n(X \times S^k) \cong H_n(X) \oplus H_{n-k}(X) \) for all \( n \geq k \).

Homework exercise 4 5 points
Solve Exercise 2.2.28 in Hatcher:

1. Use the Mayer–Vietoris sequence to compute the homology groups of the space obtained from a torus \( S^1 \times S^1 \) by attaching a M"obius band via a homeomorphism from the boundary circle of the M"obius band to the circle \( S^1 \times \{x_0\} \) in the torus.
2. Do the same for the space obtained by attaching a M"obius band to \( \mathbb{R}P^2 \) via a homeomorphism of its boundary circle to the standard \( \mathbb{R}P^1 \subseteq \mathbb{R}P^2 \).