TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

Berlin Mathematical School

Prof. Dr. John Sullivan
Hana Kouřimská

Geometry I - WS 15

http://www.math.tu-berlin.de/geometrie/Lehre/WS15/GeometryI/

Exercise Sheet 3

Exercise 1: Sphere intersections. (4 pts)

A non-empty intersection of two planes in $\mathbb{R}^3$ is a line. Prove that a non-empty intersection of two spheres in $\mathbb{R}^3$ is a circle (considering a point as a circle of radius 0).

Exercise 2: Stereographic projection in $\mathbb{R}^3$. (5 pts)

Give a brief description and provide a sketch for each of the following questions. Feel free to use colors!

(a) What are the images of the lines of latitude (the circles centered at $\pm e_3$) and of the lines of longitude (the circles through $\pm e_3$) under stereographic projection?

(b) Now rotate the pattern on the sphere to consider the circles centered at $\pm e_1$ along with the great circles through $\pm e_1$. What are their images under stereographic projection?

(c) Consider the horizontal lines in the plane (lines of the form $y = k$). What are their preimages under stereographic projection?

Exercise 3: Stereographic projection in $\mathbb{R}^2$. (4 pts)

If $C \subset \mathbb{R}^2$ is the circle centered at p with radius r, then *inversion* in C is the map τ from $\mathbb{R}^2 \setminus \{p\}$ to itself sending any point x to the point $\tau(x)$ along the same ray from p such that $\|\tau(x) - p\| = r^2/\|x - p\|$. Now consider the stereographic projection

$$\sigma : \mathbb{S}^1 \setminus \{(0,1)\} \to \mathbb{R}, \qquad (x_1, x_2) \mapsto \frac{x_1}{1 - x_2}.$$

Find a circle $C \subset \mathbb{R}^2$ such that σ is the restriction of the inversion in C.

Exercise 4: Diagonalizing quadratic forms. (3 pts)

Consider the quadratic form $Q(\mathbf{x})$ on $\mathbb{R}^3$ defined by $Q(\mathbf{x}) = x_1x_2 + x_2x_3 + x_3x_1$. Find a linear change of coordinates $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that with respect to the coordinates $\mathbf{y} = T(\mathbf{x})$, the quadratic form is diagonal: $Q(\mathbf{x}) = \sum_{i=1}^{3} \pm y_i^2$. What is its signature?

Due: Before lecture on Tuesday, 10.11.2015.