

Differential Geometry II: Analysis and Geometry on Manifolds

Exercise Sheet 3

(Diffeomorphisms, immersions, submersions)

due 15.11.2016

Exercise 1

5 points

Show that $G_1(\mathbb{R}^3) \subset \text{Sym}(3)$ is a submanifold diffeomorphic to \mathbb{RP}^2 .

Exercise 2

5 points

Let $f: N \rightarrow M$ be a smooth immersion. Prove: If f is moreover a *topological embedding*, i.e. its restriction $f: N \rightarrow f(N)$ is a homeomorphism between N and $f(N)$ (with its subspace topology), then $f(N)$ is a smooth submanifold of M .

Exercise 3

5 points

Let $X := \mathbb{C}^2 \setminus \{0\}$. The complex projective plane is the quotient space $\mathbb{CP}^1 = X/\sim$, where the equivalence relation is given by

$$\psi \sim \tilde{\psi} :\Leftrightarrow \lambda \psi = \tilde{\psi}, \quad \lambda \in \mathbb{C}.$$

Consider $\mathbb{S}^3 \subset \mathbb{R}^4 \cong \mathbb{C}^2$, then the *Hopf fibration* is the map

$$\pi: \mathbb{S}^3 \rightarrow \mathbb{CP}^1, \quad \psi \mapsto [\psi].$$

Show: For each $p \in \mathbb{CP}^1$ the *fiber* $\pi^{-1}(\{p\})$ is a submanifold diffeomorphic to \mathbb{S}^1 .