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http://www3.math.tu-berlin.de/geometrie/Lehre/WS16/DGII/ WS 16

Differential Geometry II: Analysis and Geometry on Manifolds

Exercise Sheet 10

(Pullbacks, isometries, geodesics)

due 24.01.2017

Exercise 1 **5 points**

Consider the polar coordinate map $f\colon \{(r,\theta)\in\mathbb{R}^2 \mid r>0\}\to\mathbb{R}^2$ given by $f(r,\theta) := (r\cos\theta, r\sin\theta) = (x,y)$. Show that

$$f^*(x\,dx + y\,dy) = r\,dr \qquad \text{and} \qquad f^*(x\,dy - y\,dx) = r^2\,d\theta.$$

Exercise 2 **5 points**

Let $f\colon \mathrm{M}\to\tilde{\mathrm{M}}$ and $g\colon \tilde{\mathrm{M}}\to\hat{\mathrm{M}}$ be smooth. Show that $f^*(g^*\mathrm{T}\hat{\mathrm{M}}) \cong (g\circ f)^*\mathrm{T}\hat{\mathrm{M}}$ and

$$(g\circ f)^*\hat{\nabla} = f^*(g^*\hat{\nabla})$$

for any affine connection $\hat{\nabla}$ on $\hat{\mathrm{M}}$. Show further that, if f is an isometry between Riemannian manifolds, γ is curve in M and $\tilde{\gamma} = f\circ\gamma$, then

$$\tilde{\gamma}'' = df(\gamma'').$$

Exercise 3 **5 points**

Let M be a Riemannian manifold, $\gamma\colon I\to \mathrm{M}$ be a curve which is parametrized with constant speed, and $f\colon M\to M$ be an isometry which fixes γ, i.e. $f\circ\gamma=\gamma$. Furthermore, let

$$\ker(\mathrm{id} - d_{\gamma(t)}f) = \mathbb{R}\dot{\gamma}(t), \ \textit{for all } t.$$

Then γ is a geodesic.