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http://www3.math.tu-berlin.de/geometrie/Lehre/WS16/DGII/ WS 16

# Differential Geometry II: Analysis and Geometry on Manifolds

## Exercise Sheet 11

(Geodesics, exponential map, geodesic normal coordinates)

due 31.01.2017

**Exercise 1** **5 points**

a) Is there a Riemannian manifold $(\mathrm{M}, g)$ which has finite diameter (i.e. there is an $m$ such that all points $p, q \in \mathrm{M}$ have distance $d(p,q) < m$) and there is a geodesic of infinite length without self-intersections?

b) Find an example for a Riemannian manifold diffeomorphic to $\mathbb{R}^n$ but which has no geodesic of infinite length.

**Exercise 2** **5 points**

Show that two isometries $F_1, F_2 \colon \mathrm{M} \to \mathrm{M}$ which agree at a point $p$ and induce the same linear mapping from $T_pM$ agree on a neighborhood of $p$.

**Exercise 3** **5 points**

Let $\mathrm{M}$ be a Riemannian manifold of dimension $n$. Show that for each point $p \in \mathrm{M}$ there is a local coordinate $\varphi = (x_1, \ldots, x_n)$ at $p$ such that

$$g\big(\tfrac{\partial}{\partial x_i}, \tfrac{\partial}{\partial x_j}\big)\Big|_p = \delta_{ij}, \qquad \nabla_{\frac{\partial}{\partial x_i}} \frac{\partial}{\partial x_j}\Big|_p = 0.$$