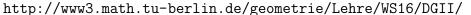
TECHNISCHE UNIVERSITÄT BERLIN Institut für Mathematik

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WS 16

Differential Geometry II: Analysis and Geometry on Manifolds

Exercise Sheet 11

(Geodesics, exponential map, geodesic normal coordinates)

due 31.01.2017

Exercise 1 5 points

- a) Is there a Riemannian manifold (M, g) which has finite diameter (i.e. there is an m such that all points $p, q \in M$ have distance d(p, q) < m) and there is a geodesic of infinite length without self-intersections?
- b) Find an example for a Riemannian manifold diffeomorphic to \mathbb{R}^n but which has no geodesic of infinite length.

Exercise 2 5 points

Show that two isometries $F_1, F_2 \colon M \to M$ which agree at a point p and induce the same linear mapping from T_pM agree on a neighborhood of p.

Exercise 3 5 points

Let M be a Riemannian manifold of dimension n. Show that for each point $p \in M$ there is a local coordinate $\varphi = (x_1, \dots, x_n)$ at p such that

$$g\left(\frac{\partial}{\partial x_i}, \frac{\partial}{\partial x_j}\right)\Big|_p = \delta_{ij}, \quad \nabla_{\frac{\partial}{\partial x_i}} \frac{\partial}{\partial x_j}\Big|_p = 0.$$