Differential Geometry II: Analysis and Geometry on Manifolds

Exercise Sheet 12

(Geodesics, Hopf-Rinow theorem)

due 07.02.2017

Exercise 1   **5 points**

A curve γ in a Riemannian manifold M is called *divergent*, if for every compact set $K \subset$ M there exists a $t_0 \in [0, a)$ such that $\gamma(t) \notin K$ for all $t > t_0$. Show: M is complete if and only if all divergent curves are of infinite length.

Exercise 2   **5 points**

Let M be a complete Riemannian manifold, which is not compact. Show that there exists a geodesic $\gamma \colon [0, \infty) \to$ M which for every $s > 0$ is the shortest path between $\gamma(0)$ and $\gamma(s)$.

Exercise 3   **5 points**

Let M be a compact Riemannian manifold. Show that M has finite diameter, and that any two points $p, q \in$ M can be joined by a geodesic of length $d(p, q)$.