TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

Mathematical Physics I, Dr. Matteo Petrera, René Zander

WS 19/20

http://www3.math.tu-berlin.de/geometrie/Lehre/WS19/MP1/

Exercise Sheet 1 (22.10.19)

Due date: **29.10.19**

- To get the Übungsschein (necessary condition for the oral exam) you need to collect *60% of the total sum of points in each half of the semester*. Each exercise sheet has 20 points. The total number of sheets is 12-14.
- Please work in fixed groups of *2 students*.
- Please justify each step of your computations. *Results without any explanation are not accepted*. Please write in a readable way. *Unreadable handwriting will not be corrected*. Feel free to write your answers either in English or in German.
- Please turn in your homework directly to me at the beginning of the Tutorial or leave it in my letter box (MA 701, Frau Jean Downes). *No homework will be accepted after the deadline has passed*.

Exercise 1 (4 pts)

1. Let $f, g\colon \mathbb{R} \to \mathbb{R}$ be two globally Lipschitz continuous functions. Prove that their composition is globally Lipschitz continuous on $\mathbb{R}$.
2. Prove that the real function
$$f(x) = \frac{1}{1+x^2}$$
is globally Lipschitz continuous.
3. Use (1) and (2) to prove that the real function
$$f(x) = \frac{1}{1+\sin^2 x}$$
is globally Lipschitz continuous.
4. Prove that the scalar IVP ($t \geq 0$)
$$\begin{cases} \dot{x} = \dfrac{e^{-t}}{1+\sin^2 x}, \\ x(0) = 1, \end{cases}$$
has a unique solution.

Exercise 2 (4 pts)

Consider the following IVP in $\mathbb{R}$:
$$\begin{cases} \dot{x} = t + x, \\ x(0) = 1. \end{cases}$$
Construct the sequence of Picard iterations and obtain the explicit solution.

Turn over

Exercise 3 (4 pts)

Consider the following IVP in $\mathbb{R}$:

$$\begin{cases} \dot{x} = |x|^{p/q}, \\ x(0) = 0, \end{cases}$$

with $p, q \in \mathbb{N} \setminus \{0\}$.

1. Prove that it has a unique solution if $p > q$.
2. Prove that it has an infinite number of solutions if $p < q$.
3. What can you say if $p = q$?

Exercise 4 (4 pts)

Consider the following IVP in $\mathbb{R}$:

$$\begin{cases} \dot{x} = \dfrac{x^2}{x^2 + \epsilon} \sqrt{|x|}, \\ x(0) = 0, \end{cases}$$

with $\epsilon > 0$. What can you say about existence and uniqueness of its solutions? Is the solution unique if $\epsilon = 0$?

Exercise 5 (4 pts)

Consider the following IVP in $\mathbb{R}$:

$$\begin{cases} \ddot{x} = 2\, t\, \dot{x} + 2\, x, \\ (x(0), \dot{x}(0)) = (1, 0). \end{cases}$$

Solve the IVP by using a power series expansion around $t = 0$.