On the Neumann problem for the Helmholtz equation in a plane angle

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We consider the Neumann boundary value problem for the Helmholtz equation in a plane angle $\beta < \pi$ and boundary data from the space $H^{-\frac{1}{2}+\varepsilon}(\Gamma)$, $0 \leq \varepsilon < 1/2$, where $\Gamma = \partial\Omega$. The case $\varepsilon = 0$ was analyzed by A. Merzon and P. Zhevandrov in 2000. Here we extend their results for $\varepsilon \in (0, 1/2)$. Namely, we prove that for these boundary conditions the solution of the Helmholtz equation in Ω exists in the Sobolev space $H^{1+\varepsilon}(\Omega)$, is unique and depends continuously on the boundary data. Moreover we give the Sommerfeld representation for these solutions. This can be used to formulate explicit compatibility conditions on the data for regularity properties of the corresponding solution. We use the method of the complex characteristics, which consists of the representation of solutions in the form of the inverse Fourier-Laplace transform of some combination of the Fourier transform of the boundary data divided by the symbol of the Helmholtz operator and using the "connection equation".

The talk is based on a joint work with A. Merzon and F.-O. Speck.