Self-adjoint fourth order differential operators with eigenvalue parameter dependent boundary conditions

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We consider the eigenvalue problem

$$y^{(4)}(\lambda, x) - (gy')'(\lambda, x) = \lambda^2 y(\lambda, x)$$

with separated boundary conditions $B_j(\lambda)y = 0$ for $j = 1, \dots, 4$, where $g \in C^1[0, a]$ is a real valued function, $B_j(\lambda)y = y^{[p_j]}(a_j)$ or $B_j(\lambda)y = y^{[p_j]}(a_j) + i\varepsilon_j \alpha \lambda y^{[q_j]}(a_j)$, $a_j = 0$ for $j = 1, 2$ and $a_j = a >$ for $j = 3, 4$, $\alpha > 0$, $\varepsilon_j \in \{-1, 1\}$. We will associate to the above eigenvalue problem a quadratic operator pencil $L(\lambda) = \lambda^2 M - i\alpha\lambda K - A$ in the space $L_2(0, a) \oplus \mathbb{C}^k$, where $M = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$ and $K = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}$ are bounded self-adjoint operators and k is the number of boundary conditions which depend on λ. We give necessary and sufficient conditions for the operator A to be self-adjoint.