The dynamics of the multiple tunnel effect

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We consider the Klein-Gordon equation on a star-shaped network composed of n half-axes connected at their origins, adding a potential which is constant but different on each branch. Our main result [3, 4] is an explicit construction of a spectral representation of the corresponding spatial operator using generalized eigenfunctions, exhibiting what we call the multiple tunnel effect.

Results in experimental physics [7, 8], theoretical physics [6] and functional analysis [1, 5] describe new phenomena created by the dynamics of the (simple) tunnel effect: the delayed reflection and advanced transmission near nodes issuing two branches. It is of major importance for the comprehension of the vibrations of networks to understand these phenomena near ramification nodes i.e. nodes with at least 3 branches, motivating our interest for the multiple tunnel effect.

Related possible applications are the $L^\infty-$time decay, the extension to coupled transmission conditions and, in the case of semi linear equations, global existence and causality (cf. [2] for the case of two branches).

References

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