Linear stochastic systems: a white noise space approach

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We present a new approach to input-output systems

$$y_n = \sum_{m=0}^{n} h_m u_{n-m}, \quad n = 0, 1, \ldots,$$

and state space equations

$$\begin{aligned} x_{n+1} &= Ax_n + Bx_n, \\ y_n &= Cx_n + Du_n, \quad n = 0, 1, \ldots \end{aligned}$$

when randomness is allowed both in the input sequence (u_n) and in the impulse response (h_n) or in the matrices A, B, C, D, which determine the state space equations. We use Hida's white noise space setting, and the Kondratiev spaces of stochastic test functions and stochastic distributions. *The key to our approach is that the pointwise product between complex numbers is now replaced by the Wick product $\lozenge$ between random variables.*Thus we have systems of the form

$$y_n = \sum_{m=0}^{n} h_m \lozenge u_{n-m}, \quad n = 0, 1, \ldots,$$

which can be shown to be (in an appropriate basis) *double convolution systems*. The Hermite transform maps the white noise space onto the reproducing kernel Hilbert space $\mathcal{F}$ with reproducing kernel $e^{\langle z,w\rangle_{\ell_2}}$ with $z, w \in \ell_2$, that is, onto the Fock space. This allows us to transfer most, if not all, problems from classical system theory into problems for analytic functions where now there is a countable number of variables. We will describe some stability theorems. The image of the Kondratiev space of distributions under the Hermite transform is a commutative ring without divisors, and we

will also explain links with the theory of linear systems over commutative rings. Finally, we will describe the parallels with double convolution systems associated to a new approach to multiscale systems, developed with M. Mboup.

The talk is based on various joint works with H. Attia, D. Levanony, M. Mboup and A. Pinhas.