Spectral and oscillation properties for fourth-order boundary value problems $Ny = \lambda Py$

J. Ben Amara

Many equations arising in elasticity theory and hydrodynamics lead to eigenvalue problems of the form

$$\tilde{P}y = \lambda \tilde{Q}y.$$

Here $\tilde{P}$ and $\tilde{Q}$ are linear operators generated by the differential expressions

$$P = (p(x)y'')'' \quad \textit{and} \quad Q = -y'' + cr(x)y,$$

respectively, where $p > 0$, $r > 0$ are continuous functions on the interval $[0, 1]$, and $c \in \mathbf{R}$ (e.g., see [1, 2, 3]). In the case of self-adjoint boundary conditions, we show that the spectrum of these Problems is real, it consists of a finite number of negative simple eigenvalues and a sequence of positive semi-simple eigenvalues tending to $+\infty$:

$$\mu_{-p} < \cdots < \mu_{-1} < 0 < \mu_1 < \mu_2 \leq \cdots \leq \mu_n \to +\infty.$$

The corresponding eigenfunction y_{-n}, $1 \leq n \leq p$, has $n-1$ zeros in $(0, 1)$, and y_1 has no zeros in $(0, 1)$. In the case when the boundary conditions are linearly depend of λ, it is shown that the negative eigenvalues of this problem (which are simple) interlace with those of the problem with self-adjoint boundary conditions. Furthermore, The corresponding eigenfunctions y_{-n} have similar oscillation properties.

References

[1] A. A. Shkalikov and C. Tretter, Spectral analysis for linear pencils $N - \lambda P$ of ordinary differential operators, Math. Nachr. 179 (1996), 275–305.

[2] H. Langer and C. Tretter, Spectral properties of the Orr-Sommerfeld problem, Proc. Roy. Soc. Edinburgh Sect. A 127, No. 6 (1997), 1245-1261.

[3] C. Tretter, Boundary eigenvalue problems for differential equation $Ny = \lambda Py$ with λ-Polynomial Boundary Conditions, J. of Differential Equations, 170, 408–471 (2001).