On negative eigenvalues of a linear pencil

T.Ya. Azizov

Let A and K be selfadjoint operators with $N(A) < \infty$ and $N(K) < \infty$, counting multiplicity, negative eigenvalues, respectively. Let $N(L)$ denote the number of negative eigenvalues of the linear pencil $L(\lambda) = A - \lambda K$. If A is a boundedly invertible operator and K is a bounded operator with a trivial kern then

$$|N(A) - N(K)| \leq N(L) \leq N(A) + N(K).$$

The main result of this talk is:

(a) if there exists a $\gamma > 0$ such that $A \geq \gamma K$ or $K \geq \gamma A$ then

$$N(L) = |N(A) - N(K)|;$$

(b) if there exists a $\gamma < 0$ such that $A \geq \gamma K$ then $N(L) = N(A) + N(K)$.

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