Canonical model theory for Hilbert space row contractions

J.A. Ball

Given any completely nonunitary Hilbert-space contraction operator T, the Sz.-Nagy model theory associates a contractive operator-valued function holomorphic on the unit disk (the characteristic function $\Theta_T(\lambda)$ of T) which has the additional property of being *pure* (there is no nonzero vector e such that $\|\Theta_T(0)e\| = \|e\|$). Conversely, any pure Schur-class function arises as the characteristic function of a completely nonunitary contraction operator. Moreover, the correspondence between completely nonunitary contraction operators T and pure Schur-class operator functions on the unit disk Θ is bijective, as long as one considers contraction operators up to unitary equivalence and Schur-class functions up to an equivalence relation known as *coincidence* (independent unitary change of basis on the input space and the output space). There have now appeared impressive extensions of this theory to multivariable settings, specifically, by Popescu to freely noncommutative row contractions, by Bhattacharyya, Eschmeier, Sarkar to commuting row contractions, and by Popescu and others to more general operator varieties. However, this model theory falls short of the completely nonunitary case and handles only the so-called completely noncoisometric case in the Sz.-Nagy-Foias theory. This talk reports on extensions of this multivariable model theory to cases beyond the completely noncoisometric case. Part of the ingredients is the the study of transfer-function realization for Schur-class functions on the noncommutative ball (contractive multianalytic functions in the sense of Popescu) and on the commutative ball (contractive multipliers of the Drury-Arveson space).

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