Truncated Toeplitz operators: existence of bounded symbols

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Truncated Toeplitz operators are compressions of usual Toeplitz operators to star-invariant (model) subspaces of H^2 in the disc: if Θ is an inner function and $K_\Theta = H^2 \ominus \Theta H^2$, then, for a function ϕ in L^2 on the circle, the truncated Toeplitz operator A_ϕ is defined by the formula $A_\phi f = P_\Theta(\phi f)$ for functions f in K_Θ such that ϕf is square integrable. Here P_Θ is the projection onto K_Θ.

A systematic study of truncated Toeplitz operators was started recently by D. Sarason. In contrast to the classical Toeplitz operators, a truncated Toeplitz operator may be sometimes extended to a bounded operator on K_Θ even for an unbounded symbol ϕ. The question, posed by Sarason, is whether boundedness of the operator implies the existence of a bounded symbol. We show that in general the answer to this question is negative. Moreover, we give a description of those inner functions Θ for which the answer is positive. In particular, we show that bounded symbols always exist in the case when Θ is a so-called one-component inner function, that is, the sublevel set $\{z : |\Theta(z)| < \varepsilon\}$ is connected for some $\varepsilon \in (0, 1)$.

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