Spectral regularity of Banach algebras and non-commutative Gelfand theory

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Let $\mathcal{B}$ be a Banach algebra with unit element. If D is a bounded Cauchy domain in the complex plane and f is an analytic $\mathcal{B}$-valued function taking invertible values on the boundary ∂D of D, the contour integral

$$\frac{1}{2\pi i}\int_{\partial D} f'(\lambda)f(\lambda)^{-1}d\lambda \tag{1}$$

is well-defined. By Cauchy's theorem, it is equal to the zero element in $\mathcal{B}$ when f has invertible values on all of D. The Banach algebra $\mathcal{B}$ is said to be *spectrally regular* if the converse of this is true. This means that (1) can only vanish in the trivial case where f takes invertible values on all of D. Matrix algebras are always spectrally regular, Banach algebras of bounded linear operators on an infinite dimensional Banach space generally not. The talk focusses on criteria for Banach algebras to be spectrally regular. These involve new aspects of non-commutative Gelfand theory using families of matrix homomorphisms.

The work reported on has been done joint with T. Ehrhardt and B. Silbermann.