The state space method in analysis and Schur complements

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In past years, notions from system theory have been used to analyze such issues in analysis as linearization and inversion of analytic operator functions, inverse Fourier transforms, the Riemann-Hilbert boundary value problem, minimal factorization of rational matrix functions, Wiener-Hopf factorization, Wiener-Hopf and Toeplitz equations, transport theory, model reduction, Szegö limits etc. A central feature in all of this is the use of *realizations*, that is of expressions of the type

$$D + C(\lambda I - A)^{-1}B \tag{1}$$

where λ is a complex variable, A, B, C, D are operators (sometimes represented by matrices) and I is an appropriate identity operator. The basis for the successful application of realizations lies in the fact that (1) turns out to be ideally suited for manipulating rational matrix functions and, more generally, analytic operator functions. The main results that demonstrate this will be reviewed briefly. Further it will be pointed out that these results can be seen as λ-versions of certain observations on Schur complements involving equivalence, extension, multiplication, inversion, and factorization. Together these observations form an algebraic framework which is of independent interest and gives rise to an intriguing open problem on Schur complements.

The talk is based on joint work done with I. Gohberg (Z"L), M.A. Kaashoek and A.C.M. Ran.