A behavioral interpretation of Livšic systems

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In the behavioral approach to (discrete-time) multidimensional linear systems, one views solution trajectories simply as the set of all solutions of a homogeneous linear system of difference equations. The duality shared by the ring of operators (polynomials with constant coefficients) and the signal space (where indeterminates act via the backward shift in their respective directions) allows one to related linear systems to commutative algebra. In this setting the transfer matrix is identified as the unique rational matrix function H satisfying $QH = P$ where $R = \begin{bmatrix} -Q & P \end{bmatrix}$ is a partitioning of the kernel representation R (i.e., the behavior is given as the kernel of the polynomial matrix R) for the behavior such that P has full column rank equal to the rank of R. This transfer matrix can be seen as a more fundamental and unifying formalism capturing the transfer functions associated with the older Givone-Roesser and Fornasini-Marchesini input/state/output approaches to multidimensional linear systems.

A quite different type of input/state/output linear system having original motivation from operator theory is the Livšic linear system, where the state-evolution equations, arising from a commuting pair of operators, are overdetermined and lead to compatibility constraints on both the input and output signals; in this case the admissible input signals are not free but form their own behavior. The main point of the present work is to identify how Livšic systems fit into the behavioral framework. In particular, we extend the transfer matrix to the setting of autonomous behaviors lacking any free variables (in which case the standard transfer matrix is trivial with no columns) by letting the reduced ring (the quotient of the polynomial ring by the annihilator of the behavior) act on the behavior. We then make explicit identifications between the transfer matrix over the reduced ring and the Livšic Joint Characteristic Function. This connection not only allows one to consider Livšic systems in a behavioral setting, but brings to light the different input/output structure that Livšic systems exhibit in comparison

to i/s/o systems with free (over the reduced ring) signals.