On the approximate K-spherical functions on the Heisenberg group

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Let G be a locally compact group, K a compact subgroup of $Aut(G)$ and dk the normalized Haar measure on K. A function $f \in \mathcal{C}(G)$ is a K-spherical function ([1]) if it satisfies the functional equation

$$\int_K f(xk \cdot y)dk = f(x)f(y), \quad x, y \in G \tag{1}$$

where $k \cdot y$ denotes the action of $k \in Aut(G)$. On the Heisenberg group $H = \mathbb{R}^2 \times \mathbb{R}$, equipped with the composition rule

$$(x, y, t)(x', y', t') = (x + x', y + y', t + t' + \frac{1}{2}(xy' - yx'),$$

if K is a two-element subgroup of $Aut(H)$, consisting of the automorphism i given by $i(x, y, t) = (y, x, -t)$ and the identity mapping, the equation (1) becomes the Stetkær functional equation ([2])

$$f(x+x', y+y', t+t'+\frac{xy' - yx'}{2})+f(x+y', y+x't-t'+\frac{xx' - yy'}{2}) = f(x, y, t)f(x', y', t'), \tag{2}$$

where $f : H \longrightarrow \mathbb{C}$ is a complex-valued function. The continuous solutions of (2) are given by Stetkær in [2]. In [3], Sinopoulos determined measurable, with respect to the first variable, solutions of this equation. In this work, we give the approximate general solutions of equation (2).

This is a joint work with E. Elqorachi.

References

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