

# On the operator-valued $K$ -spherical functions

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Let  $G$  be a locally compact group,  $K$  be a compact subgroup of  $Aut(G)$  and  $dk$  the normalized Haar measure on  $K$ . A continuous function  $f \in \mathcal{C}(G)$  is said to be  $K$ -positive definite, if for any  $n \in \mathbf{N}$ ,  $x_1, \dots, x_n \in G$  and arbitrary complex numbers  $c_1, \dots, c_n$ , we have

$$\sum_{i=1}^n \sum_{j=1}^n c_i \bar{c}_j \int_K f(x_j^{-1} k \cdot x_i) dk \geq 0. \quad (1)$$

where  $k \cdot y$  denotes the action of  $k \in Aut(G)$ . By a  $K$ -representation of  $G$ , on a Hilbert space  $\mathcal{H}$ , we mean a weakly continuous mapping  $\pi : G \longrightarrow B(\mathcal{H})$  (the algebra of all bounded operators on  $\mathcal{H}$ ) such that

- i)  $\int_K \pi(xk \cdot y) dk = \pi(x)\pi(y)$ ,  $x, y \in G$ ,
- ii)  $\pi(e) = I$  ( $I$  is the identity operator),
- iii)  $\pi(x)^* = \pi(x^{-1})$ ,  $x \in G$ , ( $\pi(x)^*$  is the adjoint operator of  $\pi(x)$ ).

In this work, we deal with the relationship between  $K$ -positive definite functions and  $K$ -representation of  $G$ , this extends classical harmonic analysis and theory of representations. Related results are given by Chojnaki in [5] and Stetkær in [6]. In [4] M. Ait Si Baha studied  $\mu$ -representations and  $\mu$ -positive definite functions, where  $\mu$  is a Gelfand measure ([1],[2],[3]).

This is a joint work with M.A. Sibaha and A. Bakali.

## References

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