# Fischer decompositions in Hermitean Clifford analysis

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A result of E. Fischer states that, given a homogeneous polynomial $q(x)$, $x \in \mathbf{R}^m$, every homogeneous polynomial $P_k(x)$ of degree $k$ can be uniquely decomposed as $P_k(x) = Q_k(x) + q(x)R(x)$, where $Q_k(x)$ is homogeneous of degree $k$, satisfying $q(D)Q_k = 0$, $D$ being the differential operator corresponding to $x$ through Fourier identification, and $R(x)$ is a homogeneous polynomial of suitable degree. If in particular $q(x) = ||x||^2$, then $q(D)$ is the Laplacian and $Q_k$ is harmonic, leading to the well–known decomposition of the spaces of complex valued homogeneous polynomials into spaces of complex valued harmonic homogeneous polynomials.

Clifford analysis is a higher dimensional function theory, constituting a refinement of harmonic analysis. It studies monogenic functions, i.e. null solutions of the rotation invariant, vector valued, first order Dirac operator $\underline{\partial}$, which factorizes the Laplacian. Fischer decompositions of the spaces of Clifford algebra valued homogeneous polynomials into spaces of monogenic homogeneous polynomials were obtained as a result. In the more recent branch Hermitean Clifford analysis, the rotational invariance has been broken by introducing a complex structure $J$ on Euclidean space and a corresponding second Dirac operator $\underline{\partial}_J$, leading to the system of equations $\underline{\partial} f = 0 = \underline{\partial}_J f$ expressing so-called Hermitean monogenicity. The invariance of this system is reduced to the unitary group. In this talk the spaces of homogeneous monogenic polynomials are further decomposed into $\mathrm{U}(n)$-irrucibles involving ho mogeneous Hermitean monogenic polynomials.

This is joint work with H. De Schepper and V. Souček.