Spectral analysis of an one – term irregular symmetric differential operator

I. Braeutigam

We consider a minimal closed symmetric operator L_0^{pq} which is induced by an irregular ordinary differential expression $l_{2m}, (m = 1, 2, \ldots)$ in $\mathcal{L}_2(I)$. We shall call the differential expression $l_{2m}[y]$ an *irregular differential expression* (see [1, ch.XIII]) if its leading coefficient vanishes at some points in I.
Let

$$l_{2m}[y](x) = (-1)^m (c(x)y^{(m)})^{(m)}(x), \quad x \in I := [-1; 1].$$

We suppose that the coefficient $c(x)$ is defined on I and has on this set only one zero $x_0 = 0$ *right-sided* order p (see [2]) and *left-sided* order q, $p, q \in \{1, 2, \ldots, 2m-1\}$, that is

$$c(x) = \begin{cases} x^p a(x), & \text{if } x \in [0; 1]; \\ (-x)^q b(x), & \text{if } x \in [-1; 0], \end{cases}$$

and functions $a(x), b(x)$ have only real values as $x \in I$ and can be represent as convergent series when $|z| < 1$

$$a(z) := a_0 + \sum_{j=1}^{+\infty} a_j z^j, \ \ a_0 \neq 0, \quad b(z) := b_0 + \sum_{j=1}^{+\infty} b_j z^j, \ \ b_0 \neq 0.$$

The deficient numbers of the operator L_0^{pq} in the upper and lower open complex planes are equal and we denote their common value as n_{pq}.

The following result holds.

Theorem 1. *The deficient numbers of the operator L_0^{pq} are defined by formula:*

$$n_{pq} = \begin{cases} 4m - \max\{p, q\}, & \text{if } p, q \in \{m+1, m+2, \ldots, 2m-1\}; \\ 2m + \min\{p, q\}, & \text{if } p, q \in \{1, 2, \ldots, m\}; \\ 3m + p - q, & \text{if } p \in \{1, 2, \ldots, m\},\ q \in \{m+1, m+2, \ldots, 2m-1\}. \end{cases}$$

Let's notice that in this case the deficient numbers of the operator are greater-than an order of the generative differential expression.

Theorem 2. *Spectrum of each self – adjoint expansion of the operator* L_0^{pq} *is discrete.*

References.

1. N. Dunford and J. T. Schwartz, Linear operators. Part II : Spectral theory. Self adjoint operators in Hilbert space, Wiley, New York 1963.

2. Yu. B. Orochko, "Deficiency indices of an one-term symmetric differential operator of an even order degenerate in the interior of an interval", Mat. Sb. 196:5 (2005), 53-82; English transl. in Sb. Math. 196:5 (2005), 673-702.