Non-commutative real algebraic geometry for a unital associative ∗-algebra

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We study non-commutative real algebraic geometry for a unital associative ∗-algebra $\mathcal{A}$ viewing the points as pairs (π, v) where π is ∗-representation of $\mathcal{A}$ on a pre-Hilbert space which contains the vector v. These general results are illustrated in the cases where $\mathcal{A}$ is one of the following ∗-algebras: $M_n(\mathbb{R})$, $M_n(\mathbb{R}[X_1, \ldots, X_d])$ and the free ∗-algebra in d variables.

For comparison, non-commutative real algebraic geometry where the points are ∗-representation of $\mathcal{A}$ - an approach of K. Schmüdgen - is well developed.