Chaotic shift operators coming from polynomials

J.A. Conejero

An operator $T \in L(X)$ is said to be hypercyclic if there exists some $x \in X$ such that $\{T^n x : n \in \mathbb{N}_0\}$ is dense in X. The first example of a hypercyclic operator on a Banach space was given by Rolewicz in 1969 [5]. He proved that the multiples λB of the backward shift operator $B(x_1, x_2, \dots) := (x_2, x_3, \dots)$ are hypercyclic on the space l_1 of absolutely summable sequences if and only if $|\lambda| > 1$. In fact, these operators are chaotic in the sense of Devaney, i.e. they are hypercyclic and have a dense set of periodic points.

Ansari proved that every power of a hypercyclic operator is also hypercyclic [1], therefore λB^n, where B^n is the n-iteration of B and $|\lambda| > 1$, it is also hypercyclic. On the other hand, Salas showed that $I + B$ is also hypercyclic [6]. The hypercyclic and chaotic behaviour of weighted backward shifts has been considered in [2, 3, 4].

Given a polynomial $p(z) = \sum_{k=0}^{n} a_k z^k$, we can consider the operator $p(B) = \sum_{k=0}^{n} a_k B^k$. We present some sufficient conditions for the hypercyclicity and chaos of these operators in terms of the coefficients of the polynomial $p(z)$.

References

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